Auctions with Frictions

Stephan Lauermann, Bonn University
Asher Wolinsky, Northwestern University

December 2020
Virtual Market Design Seminar

Conventional auction: 1 seller and N buyers with private values $v_1, ..., v_N$

Auctions as tool to study price competition in markets for assets (selling a house or a company) or services (procuring a home repair, applying at banks for a loan)

In many such scenarios, however,

- the recruitment and motivation of bidders might be a central issue (N is not exogenously given)
- the commitment ability of the auctioneer may be limited
- the interaction affected by information that auctioneer has or is trying to learn

Main feature
N is endogenous, *jointly* determined by

- seller's costly recruitment effort (marketing of a sale)
- buyer's costly entry (information acquisition costs, bid preparation costs)

Trade-off More recruitment....

- increases competition and number of high value buyers...
- ....but if more recruitment is anticipated, harder to motivate buyers to participate given costs

Findings

- Excessive recruitment & cautious bidder entry ("rat race")
- market break-down (unraveling).

Contribution
Combining recruitment and entry costs with limited commitment

Basic Setup: Auction with Bidder Solicitation

1. Seller chooses unobservable recruitment effort x; costs xs, with $s > 0$
   Number of contacted bidders is Poisson distributed with mean x
2. Contacted bidders decide whether to participate at costs $c > 0$
3. Participating bidders learn number of participants n and private value $v \in [0, 1]$ from distribution G
4. Participants submit bids, highest bidder wins and pays bid

Payoffs. If winning bid is p,

- Seller: $p - xs$
- Buyers: $v - p - c$ [winner], $-c$ [losers], 0 [non-participants]

Study symmetric (perfect Bayesian) **equilibrium**

bidding strategy $\beta(v, n) \in [0, 1]$
entry probability $q \in [0, 1]$
recruitment choice $x \geq 0$

Variations: *Other auction formats, buyer heterogeneity (prior signals/costs), unobserved participation n, uncertain seller recruitment, adverse selection, fees/subsidies/reserve price, ...* **Questions?**

Auction Stage: Bidding Equilibrium

Each participant learns

- total number of participants, n
- own value $v \in [0, 1]$, i.i.d. distributed with regular c.d.f. G

Result (Milgrom, 2004, Chapter 4)
First-price auction has unique sym. equilibrium. Denote it β_{FPA}:

1. $\beta_{FPA}(v, n) < v$ [bid shading]
2. $\beta_{FPA}(v, 1) = 0$ [monopsony]
3. $\beta_{FPA}(v, n)$ is increasing in n [oligopoly]
4. $\beta_{FPA}(v, n) \to v$ as n grows [perfect competition]

Running example: v is uniformly distributed; then,

$$\beta_{FPA}(v, n) = \frac{n-1}{n} v$$

Entry-Stage

Given recruitment effort x and participation probability q:
Expected number of (other) participants $xq =: \lambda$; Poisson(λ) distributed

Given λ and β_{FPA}, expected utility from entry is $U(\lambda)$

Observation $U(\lambda)$ is decreasing in λ, with $U(\lambda) \to 0$ for λ large

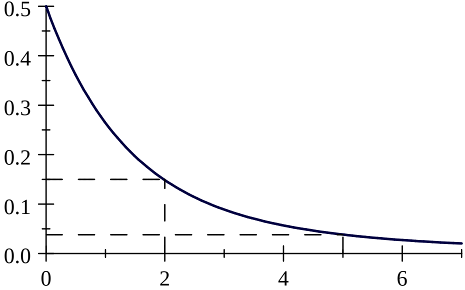
Assumption: $U(0) > c$

Break-even participation level $\lambda^B_{\max}$ solves $U\left(\lambda^B_{\max}\right) = c$

Best-Response Entry

$$
\begin{aligned}
\lambda < \lambda^B_{\max} \quad &\Rightarrow \quad q = 1 \\
\lambda = \lambda^B_{\max} \quad &\Rightarrow \quad q \in [0,1] \\
\lambda > \lambda^B_{\max} \quad &\Rightarrow \quad q = 0
\end{aligned}
$$

Expected Bidder Utility: Uniform Example



Expected Utility $U(\lambda)$.

$U(2) \approx 0.15$: For $c = 0.15$, we have $\lambda^B_{\max} \approx 2$

$U(5) \approx 0.04$: For $c = 0.04$, we have $\lambda^B_{\max} \approx 5$

Recruitment Stage [1/2]

Given β_{FPA} and expected participation λ, seller's expected revenue is $R(\lambda)$

Observation $R(\lambda)$ is increasing, $R(0) = 0$ and $R(\lambda) \to 1$ for λ large

1. Higher chance of high value bidder
2. Increasing competition increases expected bids

If seller chooses λ with recruitment effort $x = \frac{\lambda}{q}$, profit is

$$R(\lambda) - \lambda \frac{s}{q}$$

where $\frac{s}{q}$ is effective (equilibrium) cost of a bidder

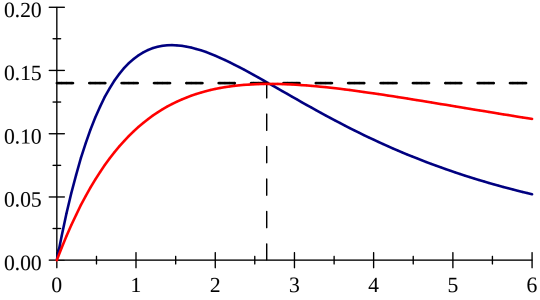
Optimum: Necessary first-order condition for interior λ,

$$R'(\lambda) = \frac{s}{q}$$

Caveat: Profit is not concave $\Rightarrow$ corner solution at 0 for high s

Break-Even Costs and Minimal-Scale

If $\frac{s}{q} > s_{\max}$, optimal participation $\lambda_s = 0$
If $\frac{s}{q} < s_{\max}$, optimal participation $\lambda_s \geq \lambda^S_{\min} > 0$ s.t. $R'(\lambda_s) = \frac{s}{q}$



BLUE: Marginal Revenue $R'(\lambda)$, RED: Average Revenue $\frac{R(\lambda)}{\lambda}$

Break-even costs $s_{\max} \approx 0.14$ and **minimal scale** $\lambda^S_{\min} \approx 2.6$

Symmetric Equilibrium Definition

Reduced Form Equilibrium $(\lambda^*, \hat{\lambda}^*, q^*, \beta^*)$:

1. Recruitment λ^* optimal given $\frac{s}{q^*}$ and β^*
2. Beliefs $\hat{\lambda}^*$ correct: If $\lambda^* > 0$, then $\hat{\lambda}^* = \lambda^*$
3. Entry decision q^* optimal given belief $\hat{\lambda}^*$ and β^*
4. Bidding behavior mutually optimal: $\beta^* = \beta_{FPA}$

If $\lambda^* = 0$ being contacted is "off-the-path":

Belief about recruitment is some $\hat{x} \geq 0$
$\Rightarrow$Belief about total participation $\hat{\lambda}^* = \hat{x} q^*$

If $q^* = 0$, then $\hat{\lambda}^* = 0$.

No-Trade Equilibrium: Market Breakdown

Proposition
If $\lambda^B_{\max} < \lambda^S_{\min}$, then there is no trade in any equilibrium for any s [bidders' break-even level is below seller's minimum scale].

Uniform example.
If $c = 0.15$, then $\lambda^B_{\max} \approx 2$, while $\lambda^S_{\min} \approx 2.6$; hence, no trade for any s.

Idea:

- Seller would like to commit to $\lambda \leq \lambda^B_{\max}$ to induce entry (for small s)
- ...but with λ unobserved, deviates to $\lambda \geq \lambda^S_{\min}$ with $\lambda^S_{\min} > \lambda^B_{\max}$

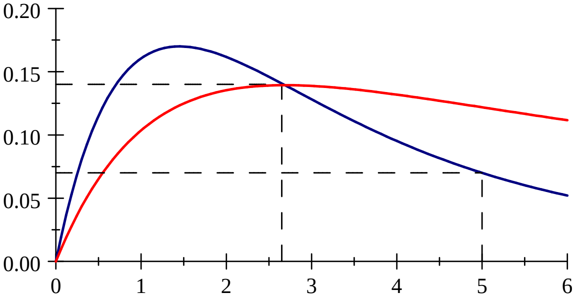
Insight:
Lack of commitment and costly entry lead to excessive recruitment; precludes trade even if recruitment is cheap.

Equilibrium has buyer beliefs $\hat{\lambda}^* = \lambda^B_{\max}$ and $q^* \in (0, 1)$ with $\frac{s}{q^*} \geq s_{\max}$.

Equilibrium with Trade

Proposition: If $\lambda^B_{\max} > \lambda^S_{\min}$, then for $\hat{s} = R'\left(\lambda^B_{\max}\right)$, maximal trade is

$$\begin{array}{lcl} s > s_{\max} & : & \lambda^* = 0 \\ \hat{s} < s < s_{\max} & : & \lambda^S_{\min} < \lambda^* < \lambda^B_{\max} \text{ with } R'(\lambda^*) = s \text{ and } q^* = 1 \\ s < \hat{s} & : & \lambda^* = \lambda^B_{\max} \text{ and } q^* \in (0,1) \text{ such that } \frac{s}{q^*} = R'\left(\lambda^B_{\max}\right) \end{array}$$



Marginal Revenue $R'(\lambda)$ [blue] and Average Revenue $\frac{R(\lambda)}{\lambda}$ [red].
If $c = 0.03$, then $\lambda^B_{\max} \approx 5 > \lambda^S_{\min}$.

Inefficiency: Excessive Recruitment

Total recruitment costs are an "equilibrium constant"

For all $s < \hat{s} = R'\left(\lambda^B_{\max}\right)$,

$$\lambda^* = \lambda^B_{\max} \text{ and } q^* \in (0,1) \text{ st. } \frac{s}{q^*} = R'\left(\lambda^B_{\max}\right)$$

Observation: Total recruitment costs are constant for all $s < \hat{s}$:

$$\lambda^* \frac{s}{q^*} = \lambda^B_{\max} R'\left(\lambda^B_{\max}\right)$$

- Cheaper recruitment (lower s) does not lower actual recruitment costs
- Inefficiency: for s small, the recruitment costs are "waste".
  - The seller would prefer to commit to $\lambda^B_{\max}$ (or even smaller than that) and both—seller and buyers—would be better off.
  - We could get approximate efficiency for small s.
- The higher $R'\left(\lambda^B_{\max}\right)$, the higher the total recruitment costs

Robustness: Other Auction Formats / Bargaining

Result independent of the auction format by revenue equivalence of standard auctions, extends to English or Dutch auction etc.

Generally:
The shape of $R(\lambda)$ and $U(\lambda)$ is what matters

Ex-ante heterogenous bidder:
Heterogeneous entry costs c or value estimates $\mathbb{E}[v]$
$\Rightarrow$Pure Equilibrium, bidders enter when c is low or value estimate high

Similar structure and qualitative insights
but not exactly constant recruitment costs etc.

Variation 1: Uncertain Seller Type

Seller's recruitment cost uncertain: $s_\ell < s_h$

In equilibrium, seller with s_ℓ samples more aggressively than s_h

Sampling bias
Contacted bidders believe the seller is likely to be the one who has sampled many others as well, inducing cautious entry (low q)

Externality
If s_ℓ is very low, then s_h is driven out of market.

Continuous distribution of recruitment costs
Seller's solicitation cost s is drawn from a smooth distribution
Example: s is uniform on $[0, s_{\max}]$
Then, for some $\hat{c}$:

- if $c < \hat{c}$, then $q^* = 1$ and all sellers choose $\lambda^*(s) > 0$, with $R'(\lambda^*(s)) = s$
- if $c > \hat{c}$, then $q^* = 0$ and all sellers choose $\lambda^*(s) \equiv 0$ for $s > 0$

Variation II: Unobservable Participation

Bidders do not observe participation n (number of competitors): bidding behavior depends on bidders beliefs $\hat{\lambda}$ and not on actual λ.

Seller has lower recruitment incentives with unobservable participation.

1. When there is trade and s is small, lower total recruitment cost. [Auction that extracts less marginal surplus good for seller.]
2. For intermediate $s \in [\hat{s}_0, s_{\max}]$, no-trade outcome ($\lambda^* = 0$) is unique. [Unravelling: Relative to optimal participation with observable participation, seller has incentive to secretly reduce recruitment.]
3. For smaller s, robust no-trade equilibrium with $\lambda^* = 0$ and $\beta^* \equiv 0$. [Bidders expect no competition and bid 0 $\Rightarrow$ recruitment unprofitable.]

Problem: Seller cannot credibly commit to generate sufficient competition.

With commitment: revenue equivalence with unobservable n
Without commitment: FPA (above) and SPA not revenue equivalent

Variation III: Quality Uncertainty

Uncertain quality of seller's object; binary example h or ℓ
Bidders have "common values": v_h or v_ℓ for all bidders
Bidders observe noisy signals about quality

Winner's curse: Winning is bad news about value.
Stronger winner's curse if more bidders participate
The more bidders, the *lower* the bids

Result
Equilibrium also has excessive recruitment, even if $c = 0$.
Total recruitment costs are constant in s.

As s decreases, seller recruits more and more.
However, bidders increasingly cautious, submit less aggressive bids.

Caveat: Equilibrium may not exist (Lauermann and Speit, 2019, "*Bidding with Uncertain Number of Competitors*").

Literature Connections

Auctions with Costly Entry (Levin&Smith, 1994, and others)

- Optimal auction design with commitment
- Main Finding: Seller can extract full surplus and chooses an efficient auction (0 reservation price; entry coordination).
- Observation: Marginal surplus of additional bidder $= U(\lambda)$

Lauermann&Wolinsky (2017,2019)

- Common value auction with informed seller
- Being recruited already contains information ("solicitation curse")
- Solicitation curse may soften price competition; inhibit price discovery

Simultaneous Search (Burdett&Judd, 1983)

- Searcher chooses a fixed sample of price
- We add asymmetric information and price quoting costs

Conclusion

Auction with endogenous participation, jointly determined by

- seller's costly recruitment effort ("marketing")
- buyer's costly entry ("bid preparation/evaluation")

Tension between seller's desire to induce aggressive bidding and participation

- Inefficiencies:
  - Wasteful recruitment, even if recruitment is cheap
  - Market breakdown possible
- Variations
  1. Uncertain seller costs: sampling bias introduces negative externalities
  2. Unobserved participation: lower recruitment incentives can reduce waste but also lead to too little competition (secret reduction of recruitment).
  3. Quality Uncertainty: excessive caution and winners' curse have similar implications