Innovation in Decentralized Markets

Exchange Design and Efficiency

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Virtual Market Design

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Financial Markets

- **Goal:** examine innovation in financial market design
  - Market-clearing technology
  - Financial products (ETFs, ETPs, derivatives, ...)
- **Main characteristics of markets:**
  - Imperfectly competitive
  - Fragmented/decentralized
- **This project**
  - Identify gains from innovation in fragmented markets
  - When is innovation efficient?

Models of Decentralized Markets

- **Search and matching approach** (e.g., Gale (1986), Duffie, Garleanu and Pedersen (2005), Vayanos and Weill (2008), Weill (2008), Duffie, Malamud and Manso (2009, 2011), Golosov, Lorenzoni and Tsyvinski (2009), Lagos and Rocheteau (2009), Lagos, Rocheteau and Weill (2011), Alfonso and Lagos (2012), Chang and Zhang (2016), Farboodi, Jarosch and Shimer (2017), Farboodi, Jarosch and Menzio (2017), Gofman (2016), Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2017, 2018), Wang (2017), Colliard, Foucault, and Hoffmann (2018), Hugonnier, Lester, and Weill (2020),...)
- **Networks approach** (e.g., Biais (1993), Kranton and Minehart (2001), Gale and Kariv (2007), Blume, Easley, Kleinberg and Tardos (2009), Manea (2011), Nava (2011), Elliott, Golub, and Jackson (2014), Babus and Kondor (2018), Condorelli and Galeotti (2016), Elliott (2014), Fainmesser (2012), Bramoulle, Kranton and D'Amours (2013), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), Choi, Galeottti and Goyal (2017), Babus and Parlatore (2018), Opp and Glode (2016) ...)
- **This project:** Imperfectly competitive markets

Model: Market

- Uniform-price double auction[1]
- **Assets:** K risky assets, indexed by k, with payoffs $\mathcal{N}(\mathbf{d}, \mathbf{\Sigma})$
- **Traders:** I agents, indexed by i who maximize

$$u(\mathbf{q}^i) = \mathbf{d} \cdot \left(\mathbf{q}_0^i + \mathbf{q}^i\right) - \frac{\alpha^i}{2} \left(\mathbf{q}_0^i + \mathbf{q}^i\right) \cdot \mathbf{\Sigma} \left(\mathbf{q}_0^i + \mathbf{q}^i\right)$$

$q_{0,k}^i = q_{0,k}^{cv} + q_{0,k}^{i,pv}$, $q_{0,k}^{i,pv}$ distributed $\mathcal{N}(E[q_{0,k}^i], \sigma_{pv}^2)$, independent across i, k

- Trader i initially holds $\mathbf{q}_0^i = (q_{0,k}^i)_k \in \mathbb{R}^K$ and trades $\mathbf{q}^i = (q_k^i)_k \in \mathbb{R}^K$
- **Market:** A market is **centralized** if there is a single market clearing for all traders and assets and **decentralized** otherwise.

[1] (e.g., Wilson (1979), Klemperer and Meyer (1989), Kyle (1989), Vayanos (1999), Vives (2011), Rostek and Weretka (2012, 2015), Ausubel et al. (2014), Sannikov and Skrzypacz (2016), Babus and Kondor (2017), Babus and Parlatore (2017), Du and Zhu (2017a,b), Kyle, Obizhaeva, and Wang (2017), Kyle and Lee (2017), Malamud and Rostek (2017), Duffie (2018), Zhang (2019), Antill and Duffie (2020), Babus and Hachem (2020a,b), Bergeman, Heumann, and Morris (2020), Chen and Duffie (2020), Chen and Zhang (2020), Manzano and Vives (2020), Wittwer (2020), Yoon (2020); see, e.g., the survey by Rostek and Yoon (2021))

Model: Market

Centralized market:

(1) **Complete participation** (w.r.t. traders and assets)

Model: Market

Centralized market:

(1) **Complete participation** (w.r.t. traders and assets)

(2) **Complete conditioning** (of (net) demands)

- Contingent schedules:

$$q_k^i(p_1, \cdots, p_K) : \mathbb{R}^K \to \mathbb{R} \qquad \forall k \in K$$

- Uncontingent schedules:[2]

$$q_k^i(p_k) : \mathbb{R} \to \mathbb{R} \qquad \forall k \in K$$

Decentralized market: (in this paper)

- Agents trade assets in K exchanges
- Each exchange is for one asset $k \in K$ (for now) and all traders I

[2] Uncontingent schedules also in Cespa (2004), Chen and Duffie (2020), Wittwer (2020).

Model: Market Clearing

- In each exchange: uniform-price double auction
  - With uncontingent schedules, the market clears *exchange-by-exchange*: $\sum_i q_k^i(p_k) = 0$, determines equilibrium p_k for each k
  - With contingent schedules, the market clears *simultaneously*: $\sum_i \mathbf{q}^{i,c}(p_1, \cdots, p_K) = 0 \in \mathbb{R}^K$ determines equilibrium price vector
  - With either type of schedule, trader i receives trade $\{q_k^i\}_k$ and pays $\sum_k p_k q_k^i$
- **Linear (Bayesian Nash) Equilibria**
- All traders are strategic (in particular, there are no noise traders)

Equilibrium in a Centralized Market

- Optimization problem in (net) demand functions of trader i:

$$\max_{\mathbf{q}^i(\mathbf{p}):\mathbb{R}^K \to \mathbb{R}^K} E[\mathbf{d} \cdot (\mathbf{q}^i + \mathbf{q}_0^i) - \frac{\alpha^i}{2}(\mathbf{q}^i + \mathbf{q}_0^i) \cdot \mathbf{\Sigma}(\mathbf{q}^i + \mathbf{q}_0^i) - \mathbf{p} \cdot \mathbf{q}^i | \mathbf{q}_0^i],$$

given other traders' demands $\{\mathbf{q}^j(\cdot)\}_{j \neq i}$ and the market clearing condition.

- **Pointwise optimization** problem of trader i:

$$\max_{\mathbf{q}^i \in \mathbb{R}^K} E[\mathbf{d} \cdot (\mathbf{q}^i + \mathbf{q}_0^i) - \frac{\alpha^i}{2}(\mathbf{q}^i + \mathbf{q}_0^i) \cdot \mathbf{\Sigma}(\mathbf{q}^i + \mathbf{q}_0^i) - \mathbf{p} \cdot \mathbf{q}^i | \mathbf{p}, \mathbf{q}_0^i] \qquad \forall \mathbf{p} \in \mathbb{R}^K$$

given the residual supply $\mathbf{S}^{-i}(\cdot) \equiv -\sum_{j \neq i} \mathbf{q}^j(\cdot) : \mathbb{R}^K \to \mathbb{R}^K$.

Equilibrium in a Centralized Market

- A demand profile $\{\mathbf{q}^i(\mathbf{p})\}_i$ is a (linear) BNE if and only if for each i:

  (i) Optimization by trader i:

$$\mathbf{d} - \alpha^i \mathbf{\Sigma}(\mathbf{q}_0^i + \mathbf{q}^i) = \mathbf{p} + \mathbf{\Lambda}^i \mathbf{q}^i \qquad \forall \mathbf{p} \in \mathbb{R}^K$$

where $\mathbf{\Lambda}^i \equiv \frac{d\mathbf{p}}{d\mathbf{q}^i} = (\frac{dp_l}{dq_k^i})_{k,l} = \begin{bmatrix} \frac{dp_1}{dq_1^i} & \cdots & \frac{dp_K}{dq_1^i} \\ \vdots & \ddots & \vdots \\ \frac{dp_1}{dq_K^i} & \cdots & \frac{dp_K}{dq_K^i} \end{bmatrix}$; hence, trader i submits:

$$\mathbf{q}^i(\mathbf{p}, \mathbf{\Lambda}^i) \;=\; (\alpha^i \mathbf{\Sigma} \;+ \mathbf{\Lambda}^i)^{-1}(\mathbf{d} - \mathbf{p} - \alpha^i \mathbf{\Sigma} \mathbf{q}_0^i) \qquad \forall \mathbf{p} \in \mathbb{R}^K;$$

  (ii) price impact of trader i is characterized by:

$$\mathbf{\Lambda}^i = -\Big(\sum_{j \neq i} \frac{\partial \mathbf{q}^j(\cdot)}{\partial \mathbf{p}}\Big)^{-1} = \Big(\sum_{j \neq i} (\alpha^j \mathbf{\Sigma} + \mathbf{\Lambda}^j)^{-1}\Big)^{-1}.$$

- If $I \to \infty$, then $\mathbf{\Lambda}^i \to 0$ (competitive market)
  If $I < \infty$, then $\mathbf{\Lambda}^i > 0$ (imperfectly competitive market)

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  (ii) price impact of trader i is characterized by

$$\boldsymbol{\Lambda}^i = \Big(\sum_{j \neq i} (\alpha^j \boldsymbol{\Sigma} + \boldsymbol{\Lambda}^j)^{-1} \Big)^{-1}, \; \boldsymbol{\Lambda}^i = \beta^i \alpha^i \boldsymbol{\Sigma}.$$

- If $I \to \infty$, then $\boldsymbol{\Lambda}^i \to 0$ (competitive market)
  If $I < \infty$, then $\boldsymbol{\Lambda}^i > 0$ (imperfectly competitive market)

Uncontingent Market: Equilibrium

- Optimization problem of trader i:

$$\max_{\{q_k^i(p_k):\mathbb{R}\to\mathbb{R}\}_k} E[\mathbf{d}\cdot(\mathbf{q}^i+\mathbf{q}_0^i)-\frac{\alpha^i}{2}(\mathbf{q}^i+\mathbf{q}_0^i)\cdot\mathbf{\Sigma}(\mathbf{q}^i+\mathbf{q}_0^i)-\mathbf{p}\cdot\mathbf{q}^i|\mathbf{q}_0^i],$$

given other traders demands $\{\{q_k^j(\cdot)\}_k\}_{j\neq i}$ and the market clearing condition.

- **Asset by asset pointwise optimization** problem of trader i:

$$\max_{q_k^i\in\mathbb{R}} E[\mathbf{d}\cdot(\mathbf{q}^i+\mathbf{q}_0^i)-\frac{\alpha^i}{2}(\mathbf{q}^i+\mathbf{q}_0^i)\cdot\mathbf{\Sigma}(\mathbf{q}^i+\mathbf{q}_0^i)-\mathbf{p}\cdot\mathbf{q}^i|p_k,\mathbf{q}_0^i] \qquad \forall p_k\in\mathbb{R},$$

given the residual supply $\{S_k^{-i}(\cdot):\mathbb{R}\to\mathbb{R}\}_k$ and his own demands for other assets $\{q_l^i(\cdot):\mathbb{R}\to\mathbb{R}\}_{l\neq k}$.

Best Response Demands

F.O.C.: for all k $\in$ K and p_k:

$$d_k - \alpha^i \sigma_{kk}(q_k^i + q_{0,k}^i) - \alpha^i \sum_{l \neq k} \sigma_{kl}(E[q_l^i | p_k, \mathbf{q}_0^i] + q_{0,l}^i) = p_k + \lambda_k^i q_k^i \qquad \forall p_k \in \mathbb{R}$$

(1) Cross-asset (cross-exchange) price impact is zero.

(2) Demand for asset k depends on expected trades of assets $l \neq k$ (vs. realized trades).

- Equilibrium is not ex post
- Price impact depends on inference

Equilibrium Characterization

Theorem Demand profile $\{\{q_k^i(p_k)\}_k\}_i$ is a (linear) Nash equilibrium in a decentralized market if, and only if,

(i) each trader i submits schedules:

$$u'(q_k^i, E[q_l^i | p_k, \mathbf{q}_0^i]) = p_k + \lambda_k^i q_k^i, \quad \forall p_k \in \mathbb{R},$$

where expected trades $E[q_l^i | p_k, \mathbf{q}_0^i]$ are characterized by the projection theorem;

(ii) residual supply for asset k is correct:

$$S_k^{-i}(p_k) = -\sum_{j \neq i} q_k^j(p_k) \qquad \forall p_k \in \mathbb{R}.$$

New technique required:

(1) Price impact no longer a sufficient statistic for the residual supply

(2) Best response problem no longer a single matrix equation
Cf. with contingent demands:

$$\mathbf{d} - \alpha^i \mathbf{\Sigma}\left(\mathbf{q}^{i,c} + \mathbf{q}_0^i\right) = \mathbf{p} + \mathbf{\Lambda}^{i,c}\mathbf{q}^{i,c} \qquad \forall \mathbf{p} \in \mathbb{R}^K, \tag{1}$$

A fixed point between price impacts and expected trades across traders and assets

The fixed point in demands is equivalent to a fixed point in price impacts $\boldsymbol{\Lambda}$!

$$\mathbf{q}^i(\mathbf{p}) = \boldsymbol{a}^i - \mathbf{B}\mathbf{q}_0^i - \mathbf{C}\mathbf{p},$$

where

$$\begin{aligned}
\boldsymbol{a}^i &= \mathbf{C}\big(\mathbf{d} - (\alpha\boldsymbol{\Sigma} - \mathbf{C}^{-1}\mathbf{B})E[\bar{\mathbf{q}}_0]\big) + \big((\alpha\boldsymbol{\Sigma} + \boldsymbol{\Lambda})^{-1}\alpha\boldsymbol{\Sigma} - \mathbf{B}\big)(E[\bar{\mathbf{q}}_0] - E[\mathbf{q}_0^i]), && \text{[intercept]}\\
\mathbf{B} &= ((1-\sigma_0^2)(\alpha\boldsymbol{\Sigma} + \boldsymbol{\Lambda}) + (I-1)\sigma_0^2\boldsymbol{\Lambda}')^{-1}\alpha\boldsymbol{\Sigma}, && \text{[coefficient on } \mathbf{q}_0^i\text{]}\\
\mathbf{C}^{-1} &= \Big[(\alpha\boldsymbol{\Sigma} + \boldsymbol{\Lambda})(\mathbf{B}\mathbf{B}')\Big]_d \Big[(\mathbf{B}\mathbf{B}')\Big]_d^{-1}, && \text{[coefficient on } \mathbf{p}\text{]}\\
\boldsymbol{\Lambda} &= \frac{1}{I-1}(\mathbf{C}^{-1})'. && \text{[price impact]}
\end{aligned}$$

Price Impact

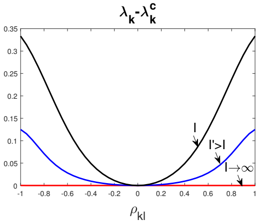
- **Price impact within exchanges** $(\{\lambda_k^i\}_k)$

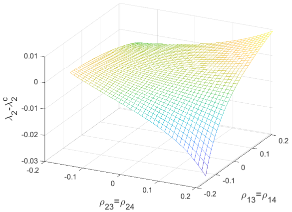
$$\lambda_k^i = -\Big(\sum_{j\neq i}\frac{\partial q_k^j}{\partial p_k}\Big)^{-1} = -\Big(\sum_{j\neq i}\underbrace{\frac{dq_k^j}{dp_k}}_{\text{direct (-)}} + \sum_{l\neq k}\underbrace{\frac{dq_k^j}{dp_l}\frac{dE[p_l|p_k]}{dp_k}}_{\substack{\text{indirect (+)}\\ \text{Cov}>0:\ (+),(+),\quad \text{Cov}<0:\ (-),\ (-)}}\Big)^{-1} \qquad (K=2)$$

- **Comparative statics of price impact** (Theorem)

  Inference effect increases price impact when $K=2$

  increases or lowers price impact when $K>2$





Welfare with Multiple Exchanges

- Independent market clearing can increase welfare
- **Example** (Preview) Let $K = 2$.

$$p_1 + \lambda^c_{11} q^{i,c}_1 + \lambda^c_{12} q^{i,c}_2 \quad \text{becomes} \quad p_1 + \lambda_1 q^i_1 + 0 q^i_2;$$
$$p_2 + \lambda^c_{21} q^{i,c}_1 + \lambda^c_{22} q^{i,c}_2 \quad \text{becomes} \quad p_2 + 0 q^i_1 + \lambda_2 q^i_2$$

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \qquad \mathbf{\Lambda}^c = \begin{bmatrix} \lambda^c_{11} & \lambda^c_{12} \\ \lambda^c_{21} & \lambda^c_{22} \end{bmatrix} \qquad \mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

- If suitably designed, markets with multiple venues are as efficient as a single exchange for any preferences and assets.
- **Why?**

  (1) Innovation of new assets.

  (2) Creation new exchanges for existing assets.

  (3) Innovation in market clearing.

  These instruments would be neutral if the market were centralized.

General Model

Let's extend the uncontingent model

- A class market structures more general in two ways:
  - **(1)** Arbitrary demand conditioning "between" contingent and uncontingent.
  - **(2)** An asset can be traded in multiple venues.
- In a market with I traders and K assets:
  - An exchange n is defined by the subset of assets traded $K(n) \subseteq K$.
  - The market structure is described by a set of N exchanges; i.e., $N = \{K(n)\}_n$.
  - Exchanges clear independently: trader i submits a demand $q^i_{k,n}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}$ for each asset $k \in K(n)$ contingent on $\mathbf{p}_{K(n)} \equiv (p_{l,n})_{l \in K(n)} \in \mathbb{R}^{K(n)}$.
  - E.g., the uncontingent market corresponds to K exchanges $N = \{\{k\}\}_k$; the contingent market corresponds to a single exchange $N = \{K\}$.

(Non)Redundancy of New Exchanges

- A new exchange n with a subset of the K assets, $K(n) \subset K$, where $(R_l)_{l \in K(n)} \sim \mathcal{N}((d_l)_{l \in K(n)}, (\sigma_{kl})_{k,l \in K(n)})$, $\sum_{\{n|k \in K(n)\}} q^i_{0,k,n} = q^i_{0,k}$ for all i.

	Contingent	Uncontingent
$I \to \infty$	X	
$I < \infty$		

- **Centralized markets**: Trading protocols are redundant

$$\mathbf{d}^+ - \alpha^i \mathbf{\Sigma}^+ (\mathbf{q}^i_0 + \mathbf{q}^i) = \mathbf{p} + \mathbf{\Lambda}^i \mathbf{q}^i$$

$$(\alpha^i \mathbf{\Sigma}^+ + \mathbf{\Lambda}^i)\mathbf{q}^i = \mathbf{d}^+ - \mathbf{p} - \alpha^i \mathbf{\Sigma}^+ \mathbf{q}^i_0.$$

Since $\mathbf{\Lambda}^i = \beta^i \alpha^i \mathbf{\Sigma}^+$ and $\mathbf{\Sigma}^+$ is singular, there is a continuum of solutions $\mathbf{q}^i(\cdot) \in \mathbb{R}^{K+K(n)}$.

- **Multi-venue markets**: Trading protocols are not redundant

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	Contingent	Uncontingent
$I \to \infty$	X	✓ Information
$I < \infty$	X	✓ Information and Liquidity

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Since $\mathbf{\Lambda}^i = \beta^i \alpha^i \mathbf{\Sigma}^+$ and $\mathbf{\Sigma}^+$ is singular, there is a continuum of solutions $\mathbf{q}^i(\cdot) \in \mathbb{R}^{K+K(n)}$.

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Innovation in Trading Technology/Market Structure

The lack of spanning motivates:

- New trading protocols (more exchanges)
- Linking trading protocols (fewer exchanges)
- Inclusion of new assets (e.g., listing) (the same number of exchanges)

Innovation

[1] Innovation is nonredundant under general conditions on the primitives.

- **Corollary** All innovations are redundant if and only if the payoffs of all assets are either perfectly correlated or independent.

Innovation

[2] Scope for/Bounds on nonredundant exchanges.

- **Example** (Equivalence to Contingent Market)

  $N = \{\{1,2\},\{2,3\},\{3,1\}\}$

- Per-unit price impact $\mathbf{\Lambda}^{+} \to \widetilde{\mathbf{\Lambda}}^{+}$ is a $K \times K$ matrix such that

$$E[\widetilde{\mathbf{q}}^{i}] = E[\mathbf{q}^{i,c}(\widetilde{\mathbf{\Lambda}}^{+})] = (\alpha\mathbf{\Sigma} + \widetilde{\mathbf{\Lambda}}^{+})^{-1}(\mathbf{d} - E[\mathbf{p}] - \alpha\mathbf{\Sigma}E[\mathbf{q}_0^{i}]).$$

  Per-unit price impact is uniquely determined:

$$\widetilde{\mathbf{\Lambda}}^{+} = \left(\mathbf{W}(\mathbf{\Lambda}^{+})^{-1}\mathbf{W}'\right)^{-1},$$

  where $\mathbf{W} \equiv (\mathbf{W}_n)_n \in \mathbb{R}^{K\times(\sum_n K(n))}$ such that, for each exchange n, the $(l,k)^{\text{th}}$ element of $\mathbf{W}_n$ is one if l^{th} asset in exchange n is asset k and zero otherwise.

- **Theorem** Let $I < \infty$ and $K > 1$ and consider two market structures $N = \{K(n)\}_n$ and $N' = \{K(n')\}_{n'}$. Suppose $\mathbf{\Sigma}$ is non-singular and $(\sigma^2_{cv}, \sigma^2_{pv}) \to 0$ and $\sigma^2_{pv}/\sigma^2_{cv} > 0$. Traders' equilibrium payoffs are the same if and only if the per-unit price impact is the same: $\widetilde{\mathbf{\Lambda}}^{N,+} = \widetilde{\mathbf{\Lambda}}^{N',+}$.

Innovation

- **Example cont'd** (Equivalence to Contingent Market)

  $N = \{\{1,2\},\{2,3\},\{3,1\}\}$ Per-unit price impact is proportional to $\mathbf{\Sigma}$

$$\widetilde{\mathbf{\Lambda}}^{+} = \left(\mathbf{W}(\mathbf{\Lambda}^{+})^{-1}\mathbf{W}'\right)^{-1} = \mathbf{\Lambda}^{c},$$

  where

$$\mathbf{W} = \left[\begin{array}{cc|cc|cc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right]$$

- Simpler (than contingent) schedules implement the contingent allocation
- Bound on nonredundant innovation

Innovation

[3] Which innovations are nonredundant?

- **Example** (Nonredundant Exchanges and Price Impact Asymmetry)

  $N = \{\{1, 2\}, \{3\}\}$

- Exchange $\{2, 3\}$ is not redundant in market N.
- In competitive markets, exchanges that increase demand conditioning (e.g., $\{2, 3\}$) are the only type of nonredundnat innovation.

  In imperfectly competitive markets, there is additional nonredundant innovation.

- Exchange $\{1\}$ is redundant in market N if and only if $\mathbf{\Lambda}_{\{1,2\}}$ is a symmetric matrix, i.e., $\lambda_{12} = \lambda_{21}$.

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma & \sigma_{12} & \sigma\rho \\ \sigma_{12} & \sigma & \sigma\rho \\ \sigma\rho & \sigma\rho & \sigma_{33} \end{bmatrix} \qquad \lambda_{12} = \lambda_{21}.$$

Innovation

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$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \qquad \lambda_{12} \neq \lambda_{21}.$$

$$\mathbf{p}^+ = \mathbf{d}^+ - \alpha\mathbf{\Sigma}^+ E[\overline{\mathbf{q}}_0^+] + \mathbf{C}^{-1}\mathbf{B}(E[\overline{\mathbf{q}}_0^+] - \overline{\mathbf{q}}_0^+)$$

$$\mathbf{C}^{-1}\mathbf{B} = (I-1)\big(\sigma_0(I-1)(\alpha\mathbf{\Sigma}^+)^{-1} + (1-\sigma_0)(\alpha\mathbf{\Sigma}^+)^{-1}\mathbf{\Lambda}(\mathbf{\Lambda}')^{-1} + (1-\sigma_0)(\mathbf{\Lambda}')^{-1}\big)^{-1}$$

If $\mathbf{\Lambda}$ is symmetric,

$$\begin{aligned} \mathbf{C}^{-1}\mathbf{B} &= (I-1)\big((1+(I-2)\sigma_0)(\alpha\mathbf{\Sigma}^+)^{-1} + (1-\sigma_0)(\mathbf{\Lambda})^{-1}\big)^{-1} \\ &= (I-1)\alpha\mathbf{\Sigma}^+\big((1+(I-2)\sigma_0)\mathbf{\Lambda} + (1-\sigma_0)\alpha\mathbf{\Sigma}^+\big)^{-1}\mathbf{\Lambda} \end{aligned}$$

Multiple Venues: Welfare

Ex ante welfare:

$$\sum_i E[u^i - \mathbf{p}\cdot\mathbf{q}^i] = \underbrace{\sum_i E[\mathbf{d}\cdot\mathbf{q}_0^i - \frac{1}{2}\mathbf{q}_0^i\cdot\alpha\mathbf{\Sigma}\mathbf{q}_0^i]}_{\text{Welfare without trade}} + \underbrace{\sum_i (E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i])\cdot\mathbf{\Upsilon}(\widetilde{\mathbf{\Lambda}}^+)(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i])}_{\text{Equilibrium surplus from trade}}$$

$$\underbrace{+(I-1)\sigma_{pv}^2 tr\left(\alpha\mathbf{\Sigma}^+\mathbf{B} - \frac{1}{2}\mathbf{B}'\alpha\mathbf{\Sigma}^+\mathbf{B}\right)}_{\text{Welfare term due to } Var(\bar{\mathbf{q}}_0|\mathbf{q}_0^i)>0},$$

where

$$\mathbf{\Upsilon}(\widetilde{\mathbf{\Lambda}}^+) \equiv \frac{1}{2}\alpha\mathbf{\Sigma}(\alpha\mathbf{\Sigma} + (\widetilde{\mathbf{\Lambda}}^+)')^{-1}(\alpha\mathbf{\Sigma} + \widetilde{\mathbf{\Lambda}}^+ + (\widetilde{\mathbf{\Lambda}}^+)')(\alpha\mathbf{\Sigma} + \widetilde{\mathbf{\Lambda}}^+)^{-1}\alpha\mathbf{\Sigma}.$$

Welfare effects:

- Diagonal element of $\widetilde{\mathbf{\Lambda}}^+$: cost of risk sharing
- Off-diagonal element of $\widetilde{\mathbf{\Lambda}}^+$: cost of diversification
- Information loss

Corollary When $K=2$, $\widetilde{\lambda}_k^{+,c} \leq \widetilde{\lambda}_k^+$ for all k.

Multiple Venues: Welfare

[4] Implications for welfare

(1) Multi-venue markets can implement the centralized-market equilibrium

if designed appropriately

(2) Neither contingent nor uncontingent design always efficient

(3) Market characteristics matter

- Joint substitutability of asset payoffs $(\boldsymbol{\Sigma})$ and trading needs $(\{E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\}_{i,k})$ matters
- Heterogeneity can favor "intermediate" market structures

(4) (2) and (3) contrast sharply with competitive markets

(5) **Proposition** Suppose that there is no information loss: i.e., $(\sigma_{cv}^2, \sigma_{pv}^2) \to 0$ and $\sigma_{pv}^2/\sigma_{cv}^2 > 0$. Given K assets, there exists a distribution of endowments $\{\mathbf{q}_0^i\}_i$ such that the ex ante welfare is strictly larger in a market structure with multiple exchanges than a single exchange for all assets.

Derivative Products

[5] Innovate on derivatives (payoff bundles) instead of exchanges (payoff subsets)?

- A new asset $k = K+1$ with return $R_{K+1} = \omega \cdot R$, where $(R_k)_{k \leq K} \sim \mathcal{N}(d, \mathbf{\Sigma})$, $R_{K+1} \sim \mathcal{N}(\omega \cdot d, \omega' \mathbf{\Sigma} \omega)$, $\mathbf{q}^i_{0,K+1} = 0$ for all i.
- Can implement the contingent outcome with uncontingent demands.
- Can derivatives mimic equilibrium effects of innovation in trading technology?

Derivative Products

Fix I, K.

(1) (**Derivatives generally do not substitute for technology**)

Proposition Fix $\mathbf{\Sigma}$. Equilibrium with derivatives can mimic equilibrium in a market N with multiple exchanges for all endowment realizations (i.e., $\widetilde{\mathbf{\Lambda}}^{+}$ coincides) if and only if price impact in N is symmetric.

- Exchanges—but not derivatives—allow inducing asymmetric cross-asset price impact.

(2) Either exchanges or derivatives may give higher welfare, depending on the realization of $(\mathbf{q}_0^i)_i$.

(**Derivatives are riskier**)

Proposition Fix $\mathbf{\Sigma}$ and $(\mathbf{q}_0^i)_i$. The payoff distribution with derivatives is riskier than with exchanges.

Thank You