Vertical Integration with Incomplete Information

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Virtual Market Design Seminar

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Vertical integration

- Vertical integration is of longstanding interest
- Renewed attention recently (guidelines, mergers, big tech)
- Traditionally viewed favorably by authorities
  - with complete information and linear prices, eliminates double marginalization
  - but efficiencies that hinge on contract restrictions are not merger specific
  - *need to take incomplete information models to heart because, in models with complete information, two-part tariffs are simply too powerful*
- Incomplete information IO ("Triple IO")
  - get key rent-extraction/social-surplus tradeoff without contract restrictions
- Challenges with tractability for Triple-IO models of vertical integration (away from special cases)

This paper

1. Tractable model of vertical integration (VI) with incomplete information
   - building on: Cramton-Gibbons-Klemperer etc.
2. Social desirability of VI depends on market conditions
   - no presumption that VI is good or bad
   - certain degree of VI is necessary and sufficient for the first-best
   - first-best market implies first-best investments
3. Rationale and guidance for divestitures
4. Boundaries of the firms
   - align with the first-best with two firms, but typically not with more (externalities, raising rivals' costs)
5. Allow for countervailing power

Related literature

- Vertical integration
  - Spengler 1950, Vertical Merger Guidelines 2020
  - Luco-Marshall 2020, Kang-Muir 2022
  - Baker et al. 2019,
    - "Vertical mergers have become increasingly prominent and controversial in antitrust policy-making"
- Triple IO
  - Loertscher-Marx 2019, 2022; Choné-Linnemer-Vergé 2021
  - Backus-Blake-Larsen-Tadelis 2020, Larsen-Zhang 2021
- Mechanism design
  - Myerson-Satterthwaite 1983, Williams 1987
  - Cramton-Gibbons-Klemperer 1987, Che 2006, Lu-Robert 2001, Loertscher-Wasser 2019

Background

(based on: Loertscher-Marx, AER, 2022)

Vertical integration and Myerson-Satterthwaite

- Consider the Myerson-Satterthwaite problem
  - 1 buyer and 1 seller
  - value and cost drawn from independent, continuous distributions with positive densities
  - assume a vertically integrated firm can resolve internal agency problem

Loertscher-Marx 2022, Prop. 6: If the supports overlap, then vertical integration is socially beneficial.

- Proof: without integration, first-best is impossible
- Vertical integration eliminates a Myerson-Satterthwaite problem

Vertical integration as a cause of a M-S Problem

- 1 buyer and $n^S \geq 2$ sellers
- Sellers' distributions have identical supports

Loertscher-Marx 2022, Prop. 7: If the lower bound of the buyer's distribution exceeds the upper bound of the sellers' distributions, then vertical integration is socially harmful.

- Proof: first-best is possible prior to and impossible after VI:
  - integrated firm's willingness to pay is the cost of its supplier
  - VI induces a setup with one buyer and multiple suppliers, all with identical supports, for which the first-best is not possible (see e.g. Gresik-Satterthwaite, 1989)
- Thus, vertical integration is the cause of a (generalized) Myerson-Satterthwaite problem

Lessons learned and questions raised

- No basis for a presumption that vertical integration is good (or bad)
  - vertical integration can eliminate or cause a Myerson-Satterthwaite problem
  - effects depend on market conditions
  - efficiency of market is endogenous

- What happens more generally?
  - allow multiple buyers and multiple suppliers

More general settings

- Obstacles
  1. trading position of the integrated firm—buy, sell, do not trade—becomes endogenous
     - second-best mechanism becomes more complicated (Lu-Robert 2001, Loertscher-Wasser 2019)
  2. modeled as a merger of a buyer and seller, VI eliminates those firms and creates a VI firm with a new distribution
     - for Myersonian mechanism design approach to be tractable, this distribution must be one-dimensional
     - unclear what the distribution of the integrated firm should be

Approach:

- build on partnership literature
- model VI as one firm taking over another's "share"
- without changing distributions or the number of firms

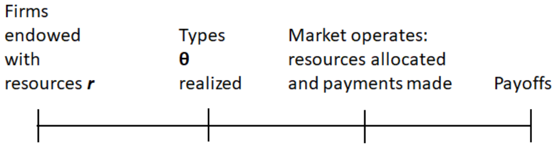
Model

Setup

- n firms: each has exclusive access to "its" downstream market but requires a common input (held by some firms)
  - trucks, spectrum, pollution permits, oil reserves, batteries
- Firm i has resources $r_i \geq 0$ and positive maximum demand $k_i \geq r_i$
  - buyer: $r_i = 0$
  - vertically integrated: $0 < r_i < k_i$
  - seller: $k_i = r_i$
- Excess demand: $R \equiv \sum_{j=1}^{n} r_j < \sum_{j=1}^{n} k_j \equiv K$
- Firm i's constant marginal value θ_i is drawn independently from a distribution with interval support and positive density

Setup (cont.)

- ***As-if approach*** to modeling the market
  - *designer* (rather than a Walrasian auctioneer)
  - market maximizes social surplus s.t. IC, IR, no deficit



  - avoid challenges of incomplete-information bargaining in extensive form
    - complicated
    - extensive form depends on market structure (and bargaining power)
- Later: transactions prior to realization of private information

Market mechanism

- Mechanism $\langle \mathbf{Q}, \mathbf{M} \rangle$ maximizes equally weighted social surplus s.t. IC, IR and no-deficit constraints
  - interim expected allocation and payment: $q_i(\theta_i)$, $m_i(\theta_i)$
  - interim expected payoff $u_i(\theta_i) \equiv q_i(\theta_i)\theta_i - m_i(\theta_i)$
- Individual rationality (IR):
  - worst-off type of firm i must not prefer its outside option $r_i\theta_i$
- A buyer's worst-off type is $\underline{\theta}$
- A seller's worst-off type is $\overline{\theta}$
- Vertically integrated firms have interior worst-off types

Conditions for the first-best to be possible

- The first-best is *possible* if there exists an IC, IR, no-deficit mechanism that achieves the first-best allocation
- Let $\Pi^e(\mathbf{r})$ be the *maximal* expected revenue of an IC, IR mechanism with a first-best allocation rule, given $\mathbf{r}$
    - maximal revenue means IR binds for the worst-off types
    - let $(\hat{\theta}_1^e, \ldots, \hat{\theta}_n^e)$ denote those worst-off types
- First-best is possible if and only if

$$\Pi^e(\mathbf{r}) \geq 0$$

    - for example, if $\Pi^e(\mathbf{r}_1) > \Pi^e(\mathbf{r}_0) \geq 0$, then there is more slack to achieve the first-best under $\mathbf{r}_1$ than under $\mathbf{r}_0$

Worst-off types

- Standard arguments (Cramton-Gibbons-Klemperer 1987) imply:

Lemma 1

Given an IC, IR mechanism $\langle \mathbf{Q}, \mathbf{M} \rangle$, firm i's worst-off type $\hat{\theta}_i$ satisfies

$$q_i(\hat{\theta}_i) = r_i.$$

- Standard mechanism design arguments: firm i's expected payoff is pinned down by q_i and $\hat{\theta}_i$ (and its virtual type function)

Socially optimal degrees of vertical integration

Vertical integration and the possibility of the first-best

- Generalization of Cramton-Gibbons-Klemperer 1987 (efficient dissolution if $\mathbf{r}$ is sufficiently symmetric) to $k_i \neq k_j$, $F_i \neq F_j$
  - instead, need worst-off types to be the same

Lemma 2 (Liu-Loertscher-Marx 2022)

There exists a unique resource vector $\mathbf{r}^*$ such that

$$\hat{\theta}_1^e(r_1^*) = \cdots = \hat{\theta}_n^e(r_n^*) \equiv \hat{\theta}^e,$$

where $\hat{\theta}^e \in (\underline{\theta}, \overline{\theta})$; moreover, $\mathbf{r}^*$ is the unique maximizer of Π^e, with

$$\Pi^e(\mathbf{r}^*) = \max_{\mathbf{r}} \Pi^e(\mathbf{r}) > 0.$$

- Observe that $\hat{\theta}^e \in (\underline{\theta}, \overline{\theta})$, which implies $r_i^* \in (0, k_i)$ for all i
  - maximizing $\Pi^e(\mathbf{r})$ requires that all firms be vertically integrated
  - an appropriate degree of vertical integration (i.e., $\mathbf{r}^*$) is sufficient for the first-best

Role of vertical integration in achieving the first-best

Proposition 2

The first-best is not possible in the absence of vertical integration and is possible when all firms are vertically integrated with endowments $\mathbf{r}^*$.

- Proof: Follows from Lemma 2 and known impossibility results for two-sided settings

  e.g., Myerson-Satterthwaite 1983, Gresik-Satterthwaite 1989, Delacrétaz et al. 2019

Effects of firm size and strength

- Connection between the first-best and $\mathbf{r}^*$ raises the question of how $\mathbf{r}^*$ varies with the size and strength of firms

Proposition 3

- With identical distributions, if $k_i > k_j$ then $r_i^* > r_j^*$.
- With $k_i = k_j$, if i's distribution first-order stochastically dominates j's, then $r_i^* > r_j^*$.

- "Larger" firms with larger maximum demands and "stronger" firms with better distributions in the sense of FOSD have more resources under $\mathbf{r}^*$

Rationale and guidance for divestitures

Effects of shifts in endowments

- Shifting endowment to a firm with a larger worst-off type weakly reduces efficiency
  - $\Pi^{\mathbf{Q}}$: expected budget surplus given $\mathbf{Q}$ and binding IR
  - $\hat{\theta}_i^{\mathbf{Q}}$: worst-off type of firm i given $\mathbf{Q}$

Proposition 6

Given IC mechanism $\langle \mathbf{Q}, \mathbf{M} \rangle$ with $\hat{\theta}_1^{\mathbf{Q}} \geq \hat{\theta}_2^{\mathbf{Q}}$ and $\Delta > 0$,

$$\Pi^{\mathbf{Q}}(r_1 + \Delta, r_2 - \Delta, r_3, \ldots, r_n) < \Pi^{\mathbf{Q}}(\mathbf{r}).$$

- Holding fixed the allocation rule, the budget surplus decreases as resources are shifted to a firm with a greater worst-off type
  - $\mathbf{Q}$ is fixed, so changes in payments relate only to the values of the outside options $r_1\hat{\theta}_1^{\mathbf{Q}}$ and $r_2\hat{\theta}_2^{\mathbf{Q}}$; shifting resources to 1 increases the total outside option of 1 and 2, which decreases their payments

Effects of shifts in endowments

- Moving beyond pairwise shifts:
  - $\mathbf{r}'$ *majorizes* $\mathbf{r}$ if for all $j \in \{1, \dots, n\}$, $\sum_{i=1}^{j} r'_{[i]} \geq \sum_{i=1}^{j} r_{[i]}$, with a strict inequality for some j and equality for $j = n$
  - if $\mathbf{r}'$ *majorizes* $\mathbf{r}$, then $\mathbf{r}'$ can be obtained from $\mathbf{r}$ by a finite number of transforms of the form described in Prop. 6 (Hardy et al. 1934, Marshall et al. 2011)

Proposition 7

Assuming identical distributions and maximum demands, given IC mechanism $\langle \mathbf{Q}, \mathbf{M} \rangle$, if $\mathbf{r}'$ majorizes $\mathbf{r}$, then $\Pi^{\mathbf{Q}}(\mathbf{r}') < \Pi^{\mathbf{Q}}(\mathbf{r})$.

Rationale and guidance for divestitures

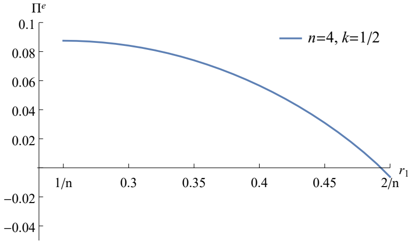
- DOJ's "2020 Merger Remedies Manual":
  - "structural remedies [divestitures] are strongly preferred in horizontal and vertical merger cases"
  - consent decree or "fix-it-first" remedy allows the authority to approve the divestiture buyer(s)

- Divestitures that move endowments towards $\mathbf{r}^*$ offer the potential for efficiency gains
- With identical distributions and maximum demands, divestitures that increase the symmetry of endowments do no harm and potentially make the first-best possible when it was not prior to the change

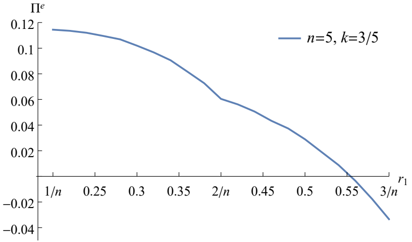
(a) $n = 4$, firm 1 acquires resources from firm 2
- eventually, $\Pi^e < 0$, so social surplus is harmed

(b) $n = 5$, firm 1 acquires the resources of firm 2 and then also acquires the resources of firm 2
- again, eventually $\Pi^e < 0$, so social surplus is harmed

(a) $\Pi^e(r_1, \frac{2}{n} - r_1, \frac{1}{n}, \frac{1}{n})$



(b) $\Pi^e(r_1, \frac{2}{n} - r_1, \frac{1}{n}, \frac{1}{n}, \frac{1}{n})$ and $\Pi^e(r_1, 0, \frac{3}{n} - r_1, \frac{1}{n}, \frac{1}{n})$



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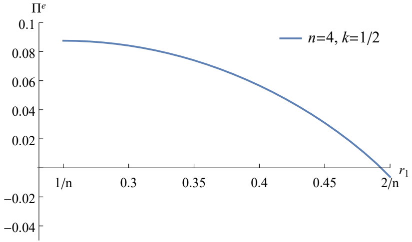
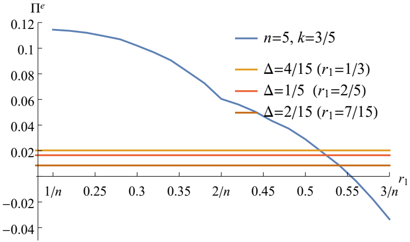


(b) Divestiture restores FB: $\Pi^e(\frac{3}{5} - \Delta, 0, 0, \frac{1}{5} + \frac{\Delta}{2}, \frac{1}{5} + \frac{\Delta}{2})$



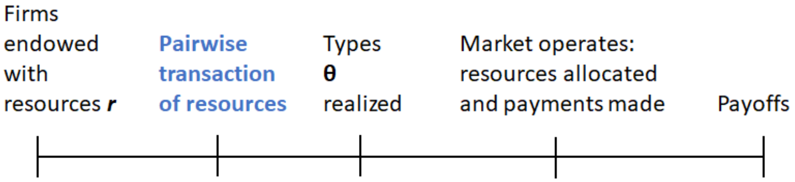
Boundaries of the Firms

Longstanding question: Boundaries of the firm?

- Determinants of firm boundaries is a longstanding question in economics
  - Why do we observe so much economic activity inside firms if markets are such powerful and effective mechanisms for allocating scarce resources? (Holmström-Roberts 1998)
- Coase 1937
  - firm boundaries explained by efficiency considerations (coordination problems, transaction costs)
- Klein-Crawford-Alchian 1978; Williamson 1975,1985; Grossman-Hart 1986; Hart-Moore 1990
  - firm boundaries explained by incentives (hold-up problem)
- Focus has been on explaining extent of vertical integration
- Externalities (raising rivals' costs): possibility of "too much" vertical integration RRC

Boundaries of the firms

- Allow resource transactions prior to type realizations
- Bilateral transactions continue as long as they are profitable



Stable endowments and the second-best

- To derive stable endowments (boundaries of the firms), need to consider **r** such that the first-best is not possible
- Need the second-best mechanism
  - Lu-Robert 2001, Loertscher-Wasser 2019
  - distort allocation away from the first-best to satisfy the no-deficit constraint
    - rank firms by their ironed weighted virtual types with weight $\frac{1}{\rho}$, where ρ is the Lagrange multiplier on the no-deficit constraint

details

Transactions consistent with profitability and the first-best

$n = 2$

Proposition 8

For $n = 2$:

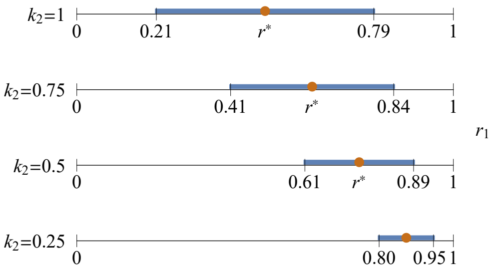
- mutually beneficial transactions exist if and only if $\Pi^e(\mathbf{r}) < 0$;
- $\mathbf{r}$ is stable if and only if it is first-best permitting.

- With $n = 2$, transactions that move the ownership structure to first-best permitting are profitable
- Once the first-best is possible, further transactions are not strictly profitable
- Boundaries of the firms align with the first-best

Illustration: Range of first-best permitting endowments

$n = 2$, $k_1 = 1$, $R = 1$, uniform $[0, 1]$ types

Range of r_1 such that the first-best is possible

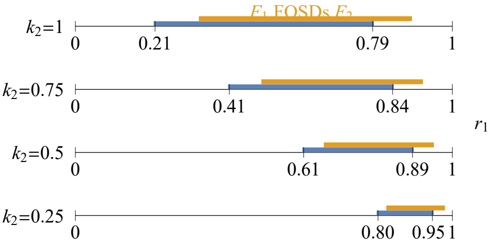


- Predicts vertically integrated firms, with bounds on extent of vertical integration
- The smaller is k_2 (less excess demand), the tighter is the stable set of endowments
- $k_2 = 1$: FB if $r_1 \in [0.21, 0.79]$ (CGK: ownership for a "dissolvable partnership")

Illustration: Range of first-best permitting endowments

$n = 2$, $k_1 = 1$, $R = 1$, 1's distribution FOSDs 2's distribution

Range of r_1 such that the first-best is possible



- Increases in firm 1's productivity shift the stable set towards higher r_1
- GM–Fisher Body interpretation:
  - GM acquired 60% of supplier Fisher Body in 1919; fully vertically integrated in 1926 (e.g., Coase 2000)
  - Model: growth in demand for GM's products (25% $\uparrow$ p.a. for Chevrolet); less for independent demand for Fisher Body (Ford $<$ 10% $\uparrow$ p.a.)
  - Larger degree of VI optimal for GM in 1926 than prior

Boundaries of the firms with $n = 3$

- Rosy picture for $n = 2$ does not carry over to $n > 3$
- Can't be in the *interior* of the first-best region
  - as long as $\Pi^e(\mathbf{r}) > 0$, there are always two firms i and j who benefit from transacting until $\Pi^e(\mathbf{r}) = 0$
- Once on the boundary, **raising rivals' costs** effects lead to the second-best
  - two transacting firms have a direct, first-order benefit from the transaction (majorization)
  - worsening of the allocation rule is, initially, second-order for them ...
  - ... and the only effect for their rivals

Boundaries of the firms relate to raising rivals' costs

Corollary 2

With $n > 2$ and $\Pi^e(\mathbf{r}) \geq 0$, vertically integrated firms have an incentive for transactions that harm rivals and reduce social surplus below the first-best.

- Suggests a role for sustained antitrust vigilance

Countervailing power

- Popular appeal and influence in antitrust of notion that equalization of bargaining power is beneficial
- Bargaining power effects viewed as distinct from productive effects
  - DOJ in AT&T–Time Warner: transaction would increase bargaining power
    - and effects from enhanced bargaining power outweighed efficiency effects
  - OECD: "Vertical integration can thus be a mechanism for an already powerful firm to further improve its bargaining position"
- Adapting the approach from Loertscher-Marx 2022, we can accommodate bargaining power effects of vertical integration

Incorporate bargaining weights

- Each firm i has bargaining weight $w_i \in [0, 1]$
  - $w_i > 0$ for at least one firm
- Market maximizes

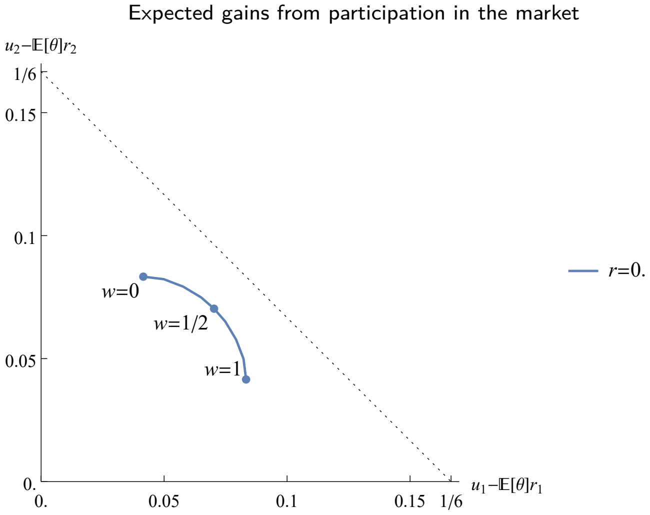
$$\sum_{i \in \mathcal{N}} w_i \mathbb{E}[\text{payoff of firm } i]$$

subject to IC, IR, and no deficit

  - allocation rule is based on the ranking of firms by their ironed weighted virtual types; weights are w_i/ρ where ρ is the Lagrange multiplier on the no-deficit constraint

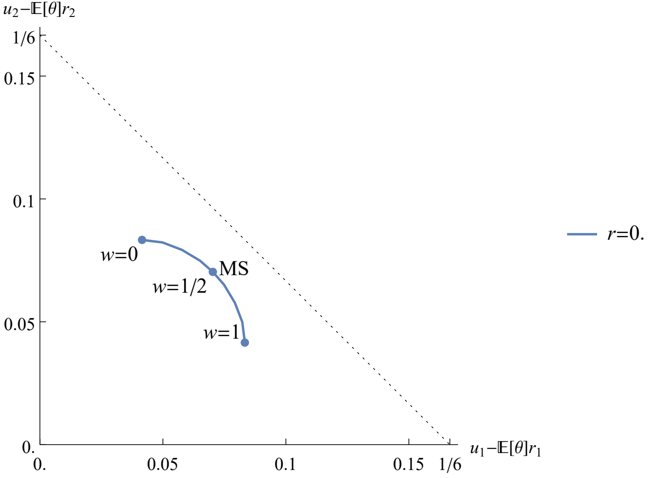
Bargaining frontier: $n = 2$, $k_1 = k_2 = 1$, $R = 1$

- Market maximizes $wu_1 + (1 - w)u_2$, where $w \in [0, 1]$



- Market maximizes $wu_1 + (1 - w)u_2$, where $w \in [0, 1]$

Expected gains from participation in the market



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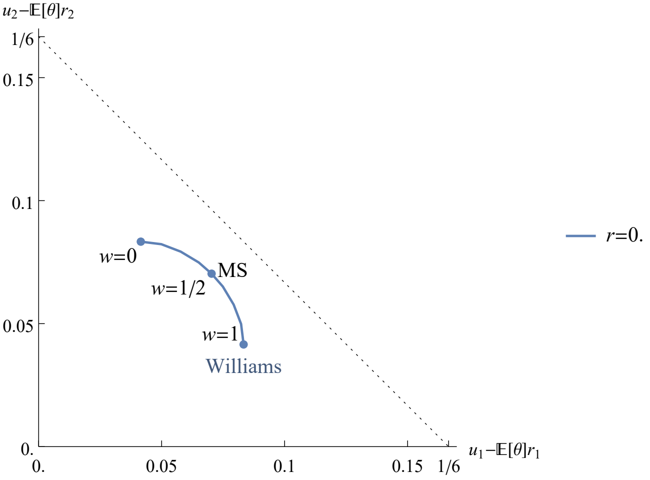

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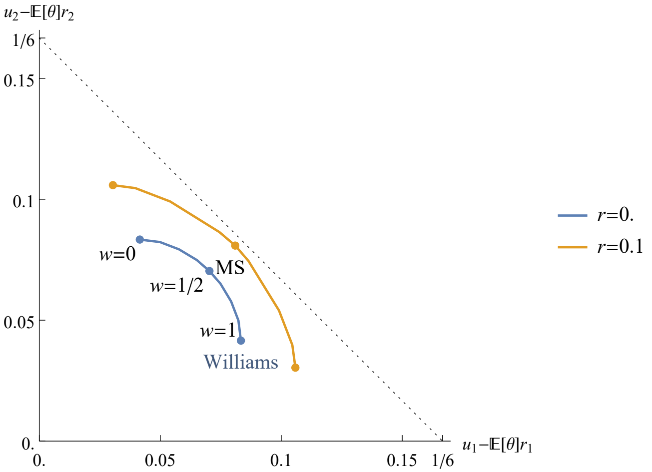

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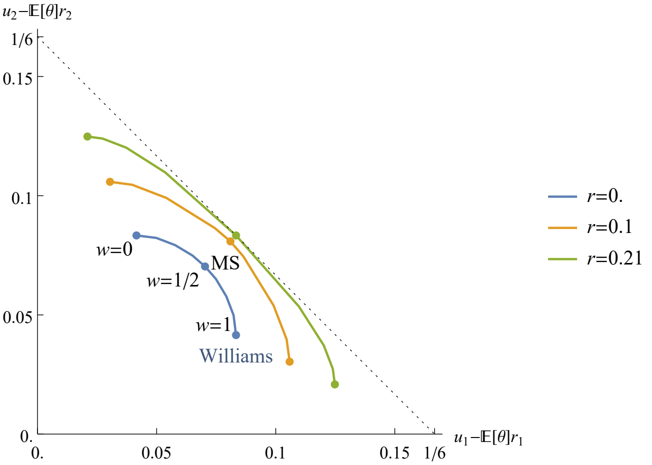

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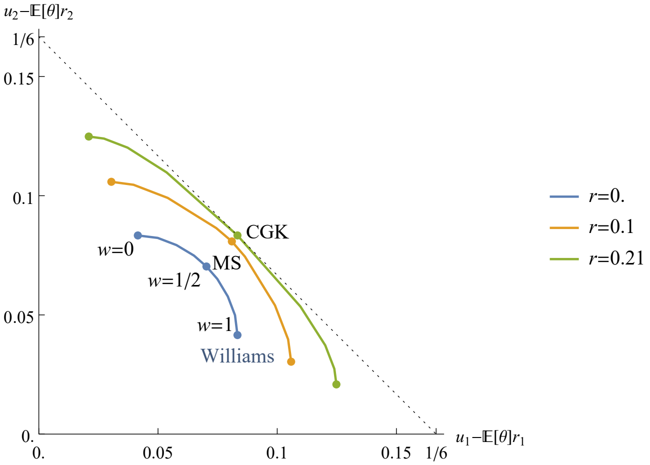

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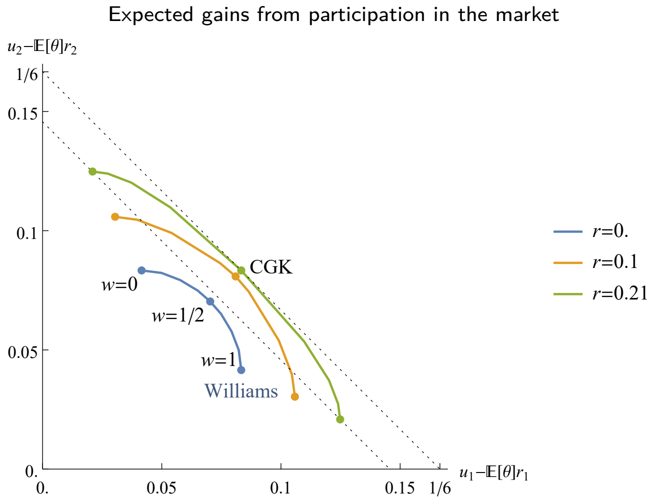

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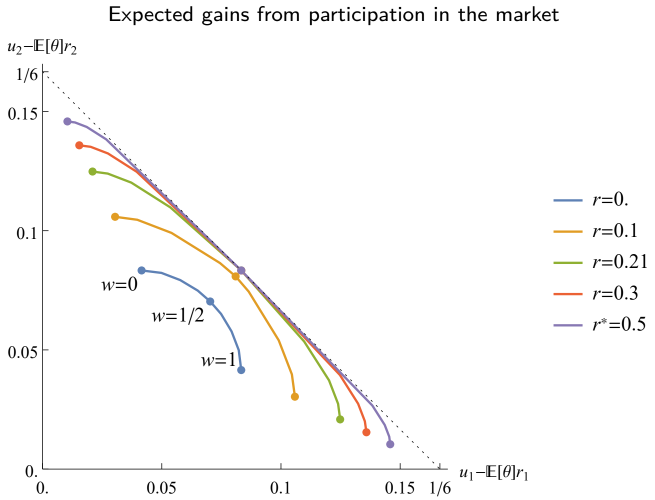
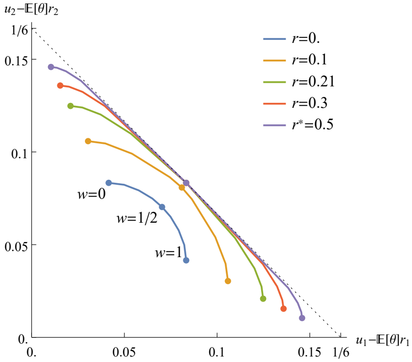


Countervailing power

- $\mathbf{r}^*$ is uniquely robust
  - $\mathbf{r}^*$ maximizes the sum of utilities, given unequal bargaining weights
  - contrasts with the indeterminacy $[0.21, 0.79]$ for equal bargaining weights
- Benefits of balanced bargaining power
  - equal bargaining weights are necessary for the first-best to be possible
- Steering: deviations from $\mathbf{r}^*$ and equal bargaining weights result in increased "steering"
  Cutler-Dafny-Grabowski-Lee-Ody 2020, Nakamura-Capps-Dranove 2007, Barro-Huckman-Kessler 2006



- firms with endowments that are "too large" ($r_i > r_i^*$) are allocated more than is efficient
- "internal sourcing" when "external sourcing" would be efficient

Extensions

Extensions

- Increasing total supply
- Investment

Conclusions

Conclusions

- Novel approach to modeling vertical integration with incomplete information
- Social surplus effects of vertical integration depend on the underlying market structure and bargaining power
  - Some extent of vertical integration is necessary for the first-best
- Provide rationale and guidance for socially beneficial divestitures
- Implications for the boundaries of the firms
  - with 2 firms, get the first-best
  - with more than 2 firms, incentives for social-surplus decreasing transactions that raise rivals' costs
- Emphasizes benefits of balanced bargaining power

Raising rivals' costs

- Raising rivals' costs theory of harm:
  - following vertical integration, the integrated firm will charge more to external buyers for the inputs that it controls
  - profitability of the strategy usually relies on diversion of downstream customers to the integrated firm
  - (Salop-Scheffman 1983,1987; Ordover-Saloner-Salop 1990; ...)
- We show that firms in the input market can be harmed by vertical integration even absent diversion considerations

return

Defining the second-best program

- If $\Pi^e(\mathbf{r}) < 0$, then the market only achieves the second-best
- Relevant Lagrangian for the problem of maximizing expected social surplus subject to IC and no deficit, assuming that IR binds, is

$$\mathcal{L} \equiv \mathbb{E}_{\boldsymbol{\theta}}\Bigg[\sum_i \overbrace{\left(\theta_i Q_i(\boldsymbol{\theta}) - (\Psi_i(\theta_i, \hat{\theta}_i) Q_i(\boldsymbol{\theta}) - \hat{\theta}_i r_i)\right)}^{\text{firm } i\text{'s surplus}}$$

$$+\rho \sum_i \Big(\overbrace{\underbrace{\Psi_i(\theta_i, \hat{\theta}_i)}_{\text{virtual type}} Q_i(\boldsymbol{\theta}) - \hat{\theta}_i r_i}^{\text{payment by firm } i}\Big)\Bigg]$$

virtual type

where ρ is the Lagrange multiplier on the no-deficit constraint

Second-best mechanism

- Given worst-off types $\hat{\boldsymbol{\theta}}$ and ρ, can solve for the allocation rule pointwise

$$Q_i^*(\boldsymbol{\theta}; \hat{\boldsymbol{\theta}}, \rho)$$

  - involves ironing for vertically integrated firms
- Using $\mathbf{Q}^*$, can define the expected budget surplus $\Pi^*(\mathbf{r}; \hat{\boldsymbol{\theta}}, \rho)$
- Solve for $\hat{\boldsymbol{\theta}}$ and ρ that deliver $q_i^*(\hat{\theta}_i) = r_i$ and $\Pi^*(\mathbf{r}) = 0$

return

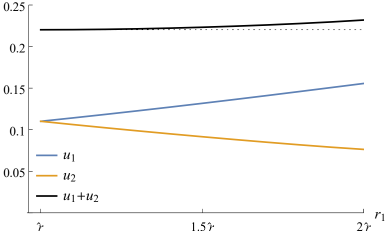
Illustration: raising rivals' costs

$n = 3,\ k_i = R = 1,\ U[0,1]$

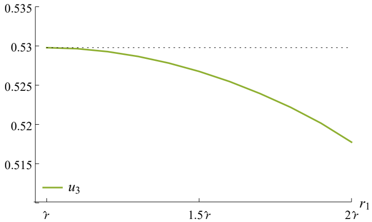
- $\hat{r}$ such that $\Pi^e(\hat{r}, \hat{r}, 1 - 2\hat{r}) = 0 \quad (\hat{r} = 0.12)$
- Allow firm 1 to acquire firm 2's endowment

$$(\hat{r} + \Delta, \hat{r} - \Delta, 1 - 2\hat{r})$$

(a) Firms 1 and 2



(b) Firm 3



- Firm 1 is better off, firm 2 is worse off (firms 1 and 2 are jointly better off)
- Firm 3, is worse off
- Social surplus decreases

return

Open issues

Open issues (1 of 2)

- Allocation of designer's budget surplus (under the first-best)
  - here we considered mechanisms that maximize equally weighted surplus
  - can introduce "bargaining weights" **w** and instead consider the mechanism that maximizes $\sum_i w_i$(firm i's surplus)
  - further, if the first-best is achieved, can have positive budget surplus $\Pi > 0$
    - can assume each firm i captures some share η_i with $\sum_i \eta_i \leq 1$
    - remainder captured by the market maker
  - a firm's bargaining weight w_i and market surplus share η_i are conceptually distinct
  - may be affected (and affected differently) by structural changes

Open issues (2 of 2)

- Mergers with both horizontal and vertical components
  - vertical: firm i acquires the resources of firm j (merged entity has resources $r_i + r_j$)
  - horizontal: merged entity has demand $k_i + k_j$ (and adjust distribution)
  - both horizontal and vertical: as above and reduces number of firms
    - adjust objective (rescale bargaining weights) and market surplus shares
- Consumer surplus effects
  - competition authority objectives may differ
  - in some settings, a focus on firm surplus is wlog because firm surplus and CS are aligned
    - resource input improves quality (known multiplicative factor)

Uniqueness of the properties of the IPV setting

- Literature identifies significant challenges from venturing outside the IPV setting
  - without private types: agents become subject to hold up—restrictions may be required to maintain tractability and/or profit–social surplus tradeoff (Mezzetti 2004, 2007)
  - without risk neutrality: optimal mechanisms depend on the nature of risk aversion, are not easily characterized, and may require payments to/from from losers (Maskin-Riley 1984, Matthews 1984)
  - without independence: no tradeoff between profit and social surplus (Cremer-McLean 1985, 1988)
  - with multi-dimensional private information and multiple agents, optimal mechanism is not known (e.g., Daskalakis et al. 2017)
  - with discrete types, no payoff equivalence theorem (mechanism not pinned down by allocation rule)
- Setup is essentially *only* one that permits a tractable approach and maintains core tradeoff between profit and social surplus

return

Mechanism details

- Mechanism $\langle \mathbf{Q}, \mathbf{M} \rangle$, type space $\mathbf{\Theta}$
- Allocation rule $\mathbf{Q} : \mathbf{\Theta} \to [0, R]^n$
  - $Q_i(\boldsymbol{\theta})$ is the quantity allocated to firm i
- Payment rule $\mathbf{M} : \mathbf{\Theta} \to \mathbb{R}^n$
  - $M_i(\boldsymbol{\theta})$ is the payment from firm i to the mechanism
- Feasibility requires that $\sum_i Q_i(\boldsymbol{\theta}) \leq R$
- No deficit: $\mathbb{E}_{\boldsymbol{\theta}}[\sum_i M_i(\boldsymbol{\theta})] \geq 0$

return

see next slide on constraints

Constraints

- Let $q_i(x)$ and $m_i(x)$ be firm i's interim expected quantity and payment if it reports x (and others report truthfully):

$$q_i(x) = \mathbb{E}_{\boldsymbol{\theta}_{-i}}[Q_i(x, \boldsymbol{\theta}_{-i})] \quad \text{and} \quad m_i(x) = \mathbb{E}_{\boldsymbol{\theta}_{-i}}[M_i(x, \boldsymbol{\theta}_{-i})]$$

- Incentive compatibility: for all θ, θ',

$$u_i(\theta) \equiv q_i(\theta)\theta - m_i(\theta) \geq q_i(\theta')\theta - m_i(\theta')$$

- Individual rationality: for all θ, $u_i(\theta) \geq r_i\theta$
- No deficit: $\mathbb{E}_{\boldsymbol{\theta}}\left[\sum_i M_i(\boldsymbol{\theta})\right] \geq 0$

return

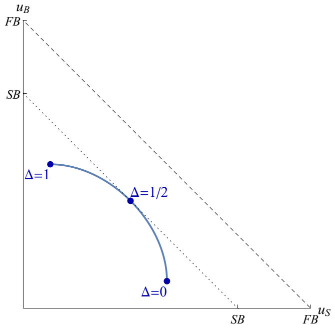
Second-best mechanism

- Use IC to write **M** in terms of **Q**
- For $\alpha \in [0,1]$, let $\mathbf{Q}_\alpha$ maximize

$$\begin{aligned} & (1-\alpha)(\text{expected SS}) \\ + \quad & \alpha(\text{expected budget surplus}) \end{aligned}$$

- Can solve for $\mathbf{Q}_\alpha$ pointwise
- $\mathbf{Q}^{SB}$ is $\mathbf{Q}_{\alpha^*}$, where α^* is the smallest α s.t. there is no deficit

First-best and second-best with 1 buyer and 1 seller



Pareto frontier for 1 single-unit buyer and 1 single-unit seller, varying weight Δ on the buyer's surplus. Types are uniformly distributed.

return

T-transform

- Given vector $(x_1, \ldots, x_n)$ and $\lambda \in (0, 1)$:
  - a T-transform of $\mathbf{x}$ is the vector with two coordinates x_j and x_k replaced by:

$$\lambda x_j + (1 - \lambda) x_k \quad \text{and} \quad \lambda x_k + (1 - \lambda) x_j$$

return

- Non-overlapping supports: $\overline{\theta}^s \leq \underline{\theta}^b$
- Firm 3 acquires the resources of firm 2
  - $\hat{r}_1 = 1$, $\hat{r}_2 = 0$, $\hat{r}_3 = 1$
- Firm 3's value for the input exceeds the wtp of others, so firm 3 effectively leaves the market
- Firm 1 is a seller and firm 2 is a buyer with identical supports:

Proposition

Given a pre-integration market with one-to-many trade and nonoverlapping supports, vertical integration—modeled as the buyer acquiring the resources of a seller—decreases social surplus.

- As before, vertical integration creates a Myerson-Satterthwaite problem

return

Virtual types

- Let F_i be the type distribution for i, with positive density f_i
- Virtual type functions for net buyers and net sellers:

$$\Psi_i^B(\theta) \equiv \theta - \frac{1 - F_i(\theta)}{f_i(\theta)} \quad \text{and} \quad \Psi_i^S(\theta) \equiv \theta + \frac{F_i(\theta)}{f_i(\theta)}.$$

- Typically assumed increasing
- Also define overall virtual type function:

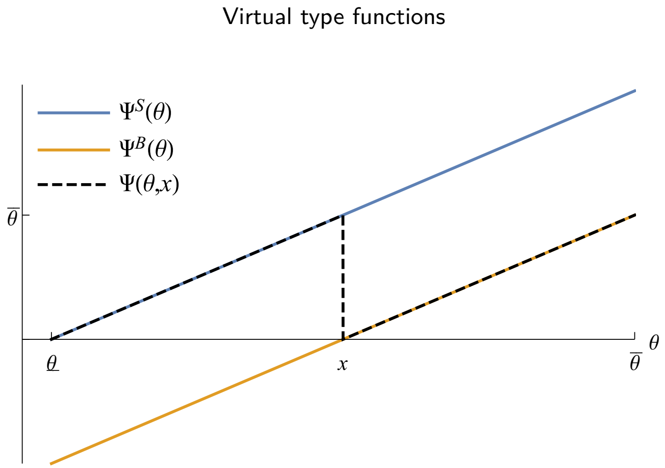
$$\Psi_i(\theta, x) \equiv \begin{cases} \Psi_i^S(\theta) & \text{if } \theta < x, \\ \Psi_i^B(\theta) & \text{otherwise,} \end{cases}$$

which is not increasing in θ if x is interior—in particular, it jumps down at $\theta = x$

return

Virtual type functions

- Example with uniformly distributed types



return

Second-best mechanism

- Define *weighted* virtual type functions:

$$\Psi^B_{i,\alpha}(\theta) \equiv \theta - (1-\alpha)\frac{1 - F_i(\theta)}{f_i(\theta)} \quad \text{and} \quad \Psi^S_{i,\alpha}(\theta) \equiv \theta + (1-\alpha)\frac{F_i(\theta)}{f_i(\theta)}$$

and the associated overall function with cutoff x:

$$\Psi_{i,\alpha}(\theta, x) \equiv \begin{cases} \Psi^S_{i,\alpha}(\theta) & \text{if } \theta < x, \\ \Psi^B_{i,\alpha}(\theta) & \text{otherwise} \end{cases}$$

- Then we can rewrite the Lagrangian as

$$\mathcal{L} \equiv \rho \mathbb{E}_{\boldsymbol{\theta}} \left[\sum_{i \in \mathcal{N}} \Psi_{i,\frac{1}{\rho}}(\theta_i, \hat{\theta}_i) Q_i(\boldsymbol{\theta}) \right] + \sum_{i \in \mathcal{N}} (1-\rho)\hat{\theta}_i r_i,$$

where $\rho \geq 1$ is the multiplier on the no-deficit constraint

Second-best mechanism

- Let $\overline{\Psi}_{i,\alpha}$ denote the ironed weighted virtual type function (see next slide)
- Given ρ, one can maximize $\mathcal{L}$ with respect to $\mathbf{Q}$ pointwise, subject to each Q_i being nondecreasing (necessary and sufficient for IC), by choosing $\mathbf{Q}$ to maximize (Loertscher-Wasser 2019)

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\sum_{i\in\mathcal{N}}\overline{\Psi}_{i,\frac{1}{\rho}}(\theta_i,\hat{\theta}_i)Q_i(\boldsymbol{\theta})\right]$$

- Specifically, rank the ironed weighted virtual types $\overline{\Psi}_{i,\frac{1}{\rho}}(\theta_i,\hat{\theta}_i)$ from largest to smallest and assign the supply to the highest-ranked agents (breaking ties with a probability such that IR is satisfied)

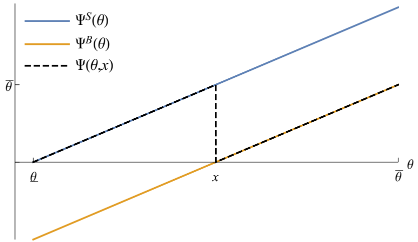
return

return cp

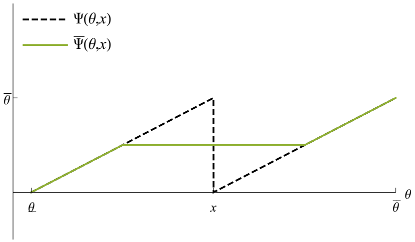
Virtual type functions

- Example with uniformly distributed types

(a) Virtual type functions



(b) Ironed virtual type function



Setup for CS effects with downstream consumers

- Each firm i is a retailer serving a downstream market with demand $D_i(p)$ when the mass of consumers is $\omega = 1$; constant marginal costs of zero
  - $p_i^* D_i(p_i^*)$ and $CS_i^* = \int_{p_i^*}^{\overline{p}} D_i(p)dp - p^* D_i(p^*)$ are maximized revenue and equilibrium consumer surplus
- With mass ω_i of consumers, which is the private information of the firm, profit and consumer surplus are

$$\omega_i p_i^* D_i(p_i^*) \quad \text{and} \quad \omega_i CS_i^*$$

- Resource input improves quality by $\delta > 0$ and thus consumer surplus by $\omega_i \delta CS_i^*$
- Hence, $v_i = \omega_i \delta p^* D_i(p^*)$ is firm i's wtp for the input
- CS increases with the probability of trade and the efficiency of the allocation among firms

return

Contract restrictions

- The incomplete information setup assumes:
  - transactions—horizontal or vertical mergers—occur before private information is realized
  - the only contracts available at that stage are simple transactions of property rights
  - in particular, no contracts are admissible that are contingent of future realizations of private information
- So this is a restriction on the contracting space at the *ex ante* stage
- Without it,
  - the first-best would always be possible
  - but it would also be hard to see how firms—that can bind themselves in these ways—are still independent entities

▸ return

Payments in the market mechanism

- Virtual type functions for net buyers and sellers:

$$\Psi_i^B(\theta) \equiv \theta - \frac{1 - F_i(\theta)}{f_i(\theta)} \quad \text{and} \quad \Psi_i^S(\theta) \equiv \theta + \frac{F_i(\theta)}{f_i(\theta)}.$$

- Overall virtual type function with cutoff type x:

$$\Psi_i(\theta, x) \equiv \begin{cases} \Psi_i^S(\theta) & \text{if } \theta < x, \\ \Psi_i^B(\theta) & \text{otherwise.} \end{cases}$$

Proposition 1

Given IC, IR mechanism $\langle \mathbf{Q}, \mathbf{M} \rangle$, firm i's expected payment to the mechanism is

$$\mathbb{E}_{\theta_i}[m_i(\theta_i)] = \mathbb{E}_{\theta_i}\left[\Psi_i(\theta_i, \hat{\theta}_i) q_i(\theta_i)\right] - u_i(\hat{\theta}_i),$$

where $u_i(\hat{\theta}_i) \geq r_i \hat{\theta}_i$ (and $u_i(\hat{\theta}_i) = r_i \hat{\theta}_i$ if IR binds for firm i).

▸ return

Increasing total supply

- Firms can have excess incentives to increase in their own resources
  - firm's willingness to buy incremental endowment exceeds planner's willingness to pay
- Within the first-best, shifts surplus away from rivals
- Can render the first-best no longer possible
  - range of first-best permitting endowments shrinks

- Contrasts with Farrell-Shapiro (1990): in an oligopoly setup, increases in the assets of the largest (lowest-cost) firm can increase welfare

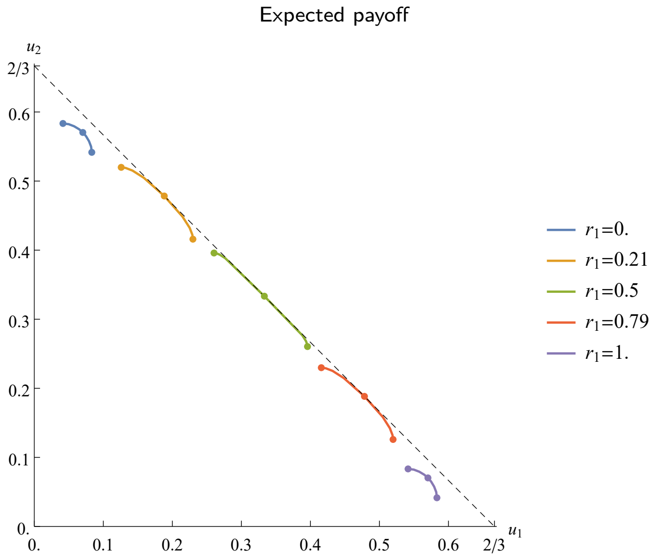
return

Investment

- Investments improve type distributions (without changing supports); not observable and not contractible
- Firms simultaneously choose investments prior to type realization and market mechanism
- Result: if the market is efficient, then there is a Nash equilibrium in which investments are the same as the planner would choose
  - no tension between efficient investments and efficient bargaining—efficient bargaining implies that there is an equilibrium with efficient investments
- Applied to GM–Fisher Body:
  - degree of vertical integration that became necessary to permit efficient transactions as the demand for GM's cars grew also aligned incentives for efficient investments in productivity

return

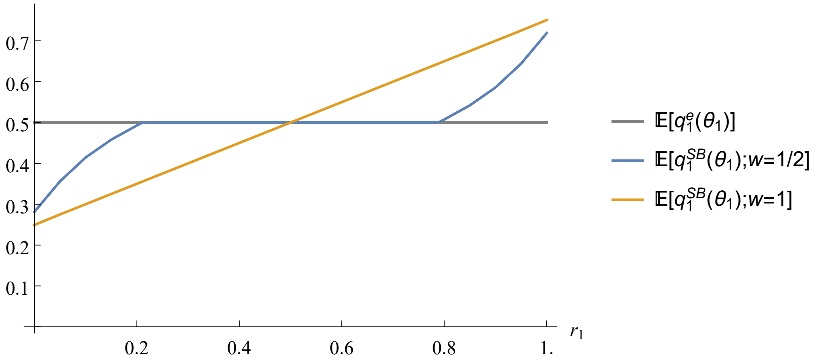
Bargaining frontier: $n = 2$, $k_1 = k_2 = 1$, $R = 1$



return

Bargaining frontier: $n = 2$, $k_1 = k_2 = 1$, $R = 1$

Expected allocation as a function of endowment and bargaining weight



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