Unpaired Kidney Exchange:

Overcoming the double coincidence of wants without money

UNDER REVISION

Virtual Market Design Seminar

Online

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March 29,2021

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IN ECONOMICS AND STATISTICS

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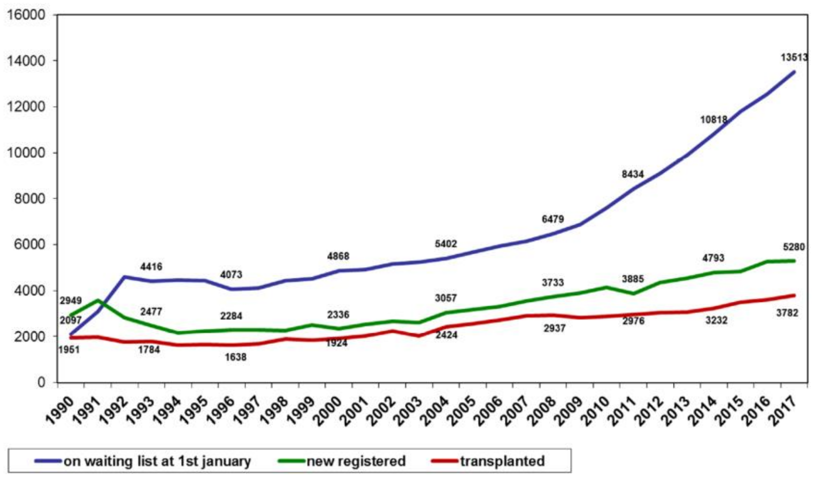
Background

- Transplantation from **Deceased** donors
  - Supply has been steady over the past 10y
  - Demand has grown: ↑ **patients on the waiting list**
    - +65% in the U.S., +100% in France (-9% in the U.K.) (2007 to 2017)



Source: Agence de la Biomédecine (France)

- Transplantation from **Living** donors
  - In average: **better quality than deceased donors**
  - Has increased steadily: 40% of all kidney transplants world. in 2015…
  - … but **many patients are *incompatible* with their donor**: ≈40% in Europe

Development of Kidney Exchange Programs

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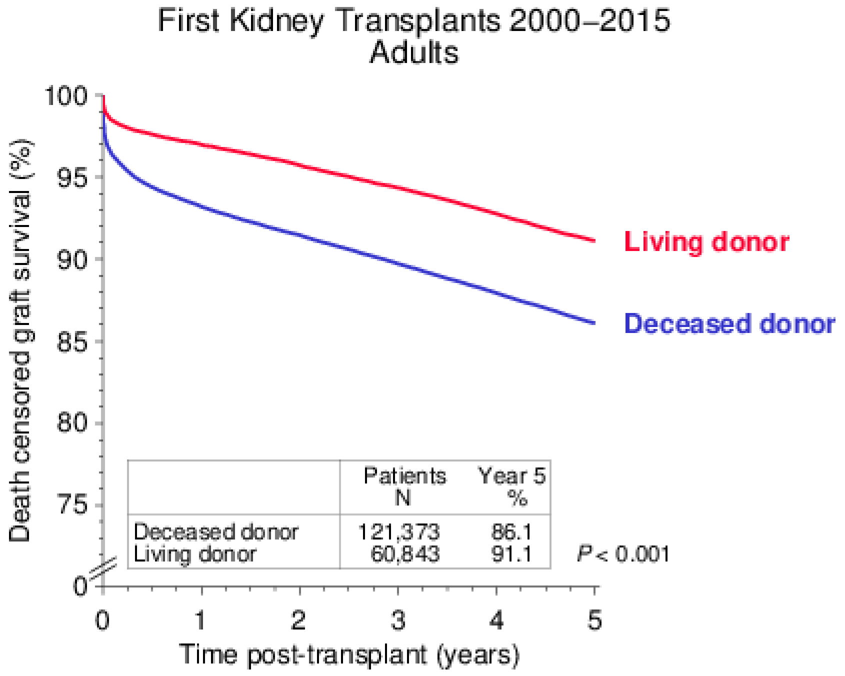

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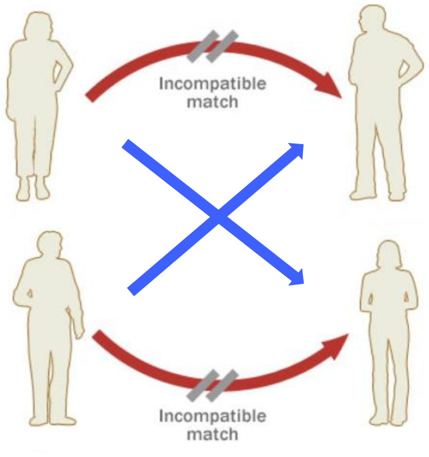
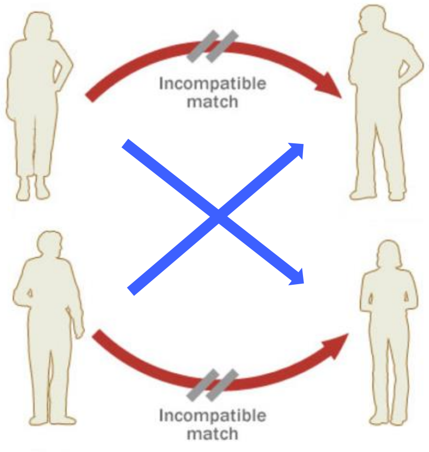
Development of Kidney Exchange Programs

First Exchange Technology for Kidneys



Pairwise
exchanges

First Exchange Technology for Kidneys



Pairwise exchanges

- ✓ Simultaneaous exchanges
- ✓ Donor gives while patient receives
- ✓ No renege risks of donors
- ✓ Widely used (UK, US, FR…)
- × Limit on the size (up to 3-4)

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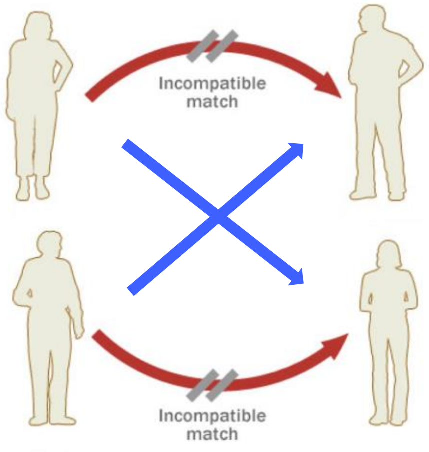

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“Double Coincidence of Wants”
(Jevons, 1885 & Roth-Sonmez-Unver, 2007)

Tackling Double Coincidence of Wants

1. Legalize kidney sales

   [*Becker-Elias (2007), Akbarpour-Fatemi-Matoorian (2019),…*]

2. Thicken the market by increasing participation // by waiting

   [*Roth-Sonmez-Unver (2007), Sonmez-Unver-Yenmez (2020) // Unver (2010), Akbarpour-Li-Gharan (2020), I Ashlagi-Nikzad-Strack (2019),…*]

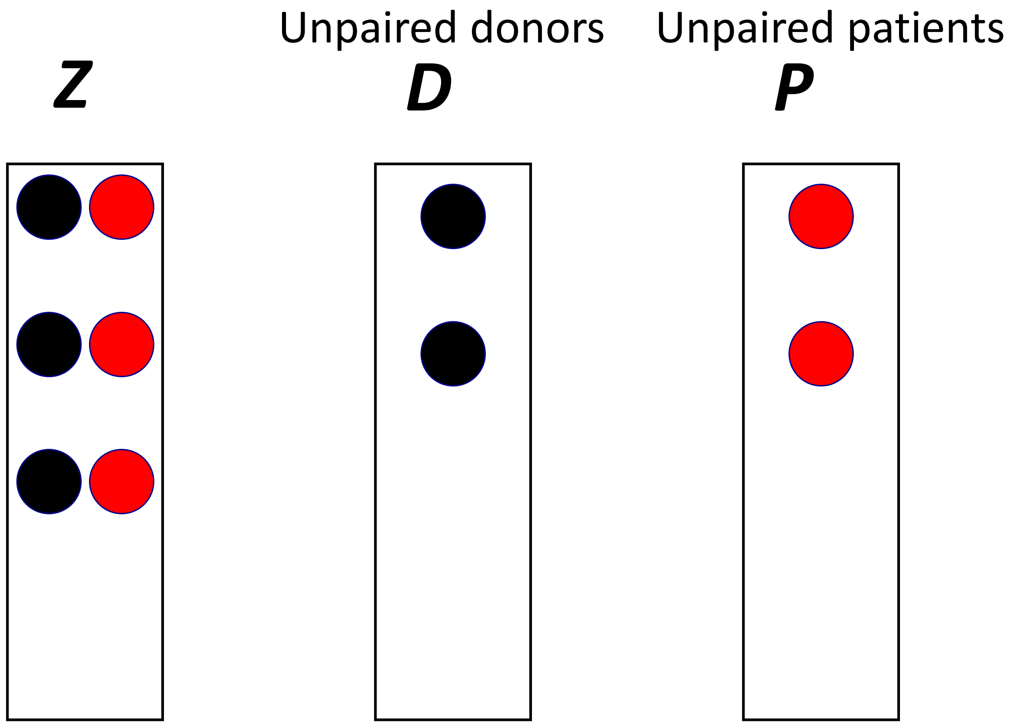
3. Altruistic donor chains

   [*Roth-Sonmez-Unver-Delmonico-Saidman (2006), Ashlagi-Gamarnik-Rees-Roth (2012), Anderson-Ashlagi-Gamarnik-Kanoria (2017), Ashlagi-Burq-Jaillet-Manshadi (2019),…*]

4. Sequential kidney exchange and vouchers

   [*Ausubel-Morrill (2014), Veale et al. (2017),…*]

Unpaired Kidney Exchange



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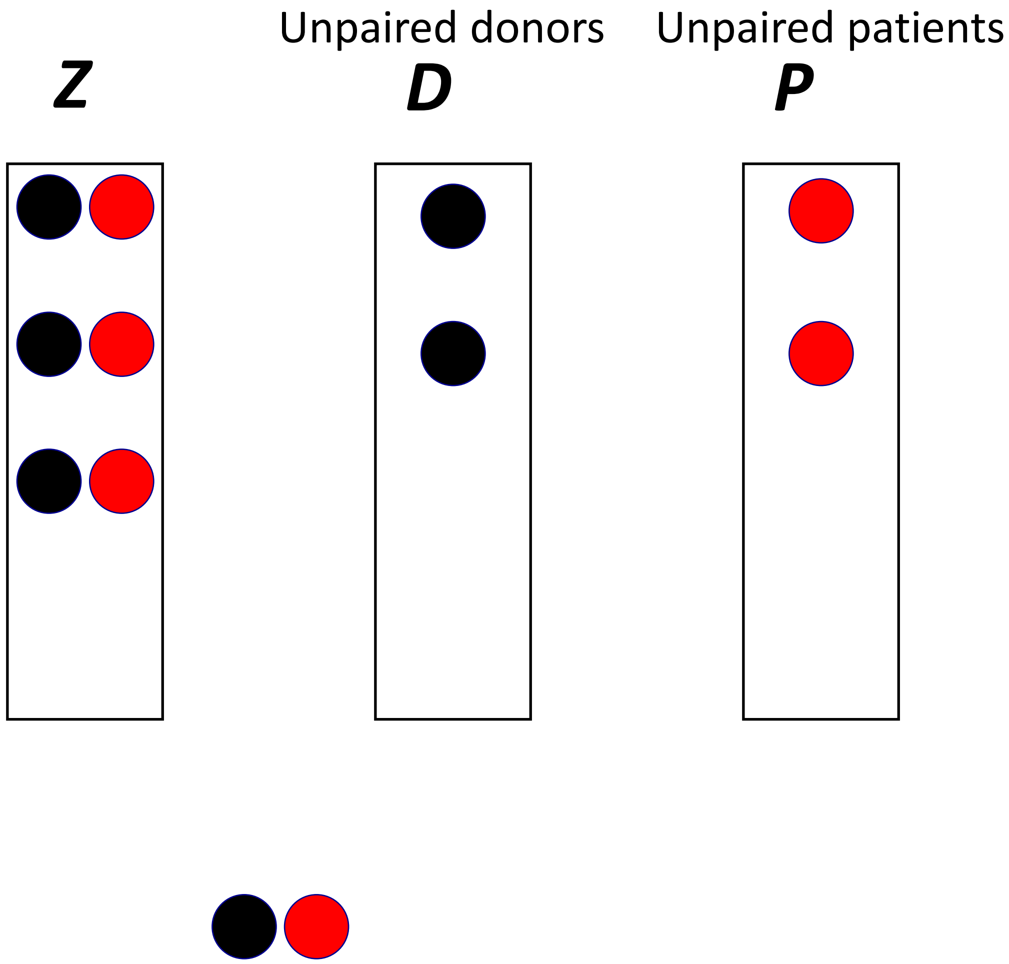

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- Check if the patient can receive from a donor in ***D*** *or* ***Z***
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1. Donor in ***D*** preferred to donor in ***Z*** and patient in ***P*** over patient in ***Z***
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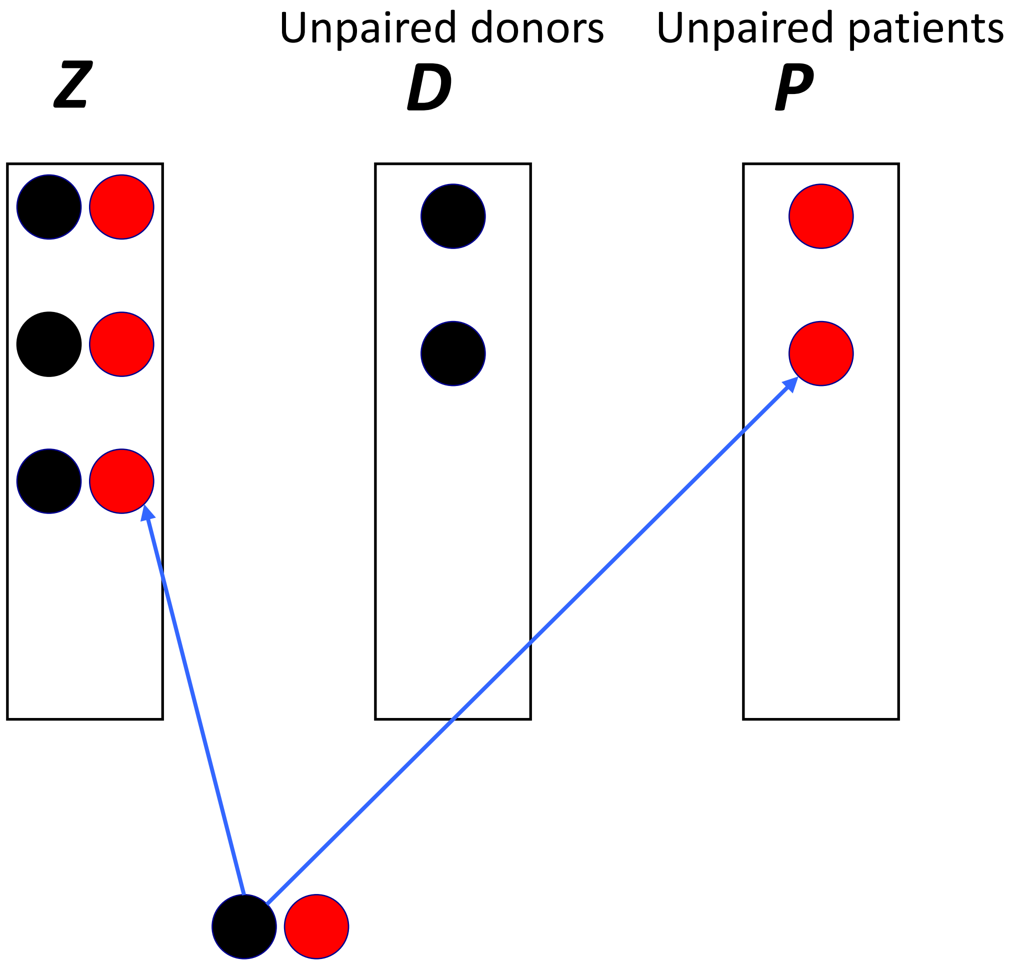

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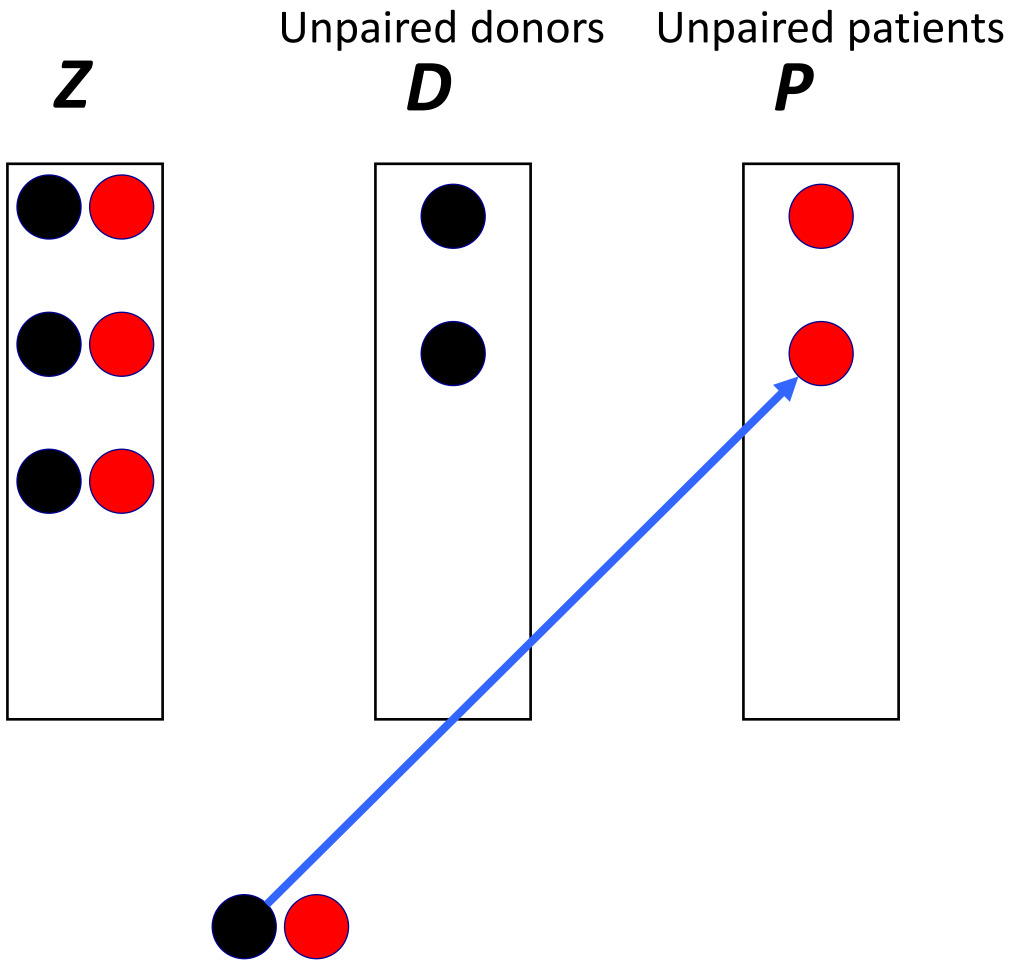

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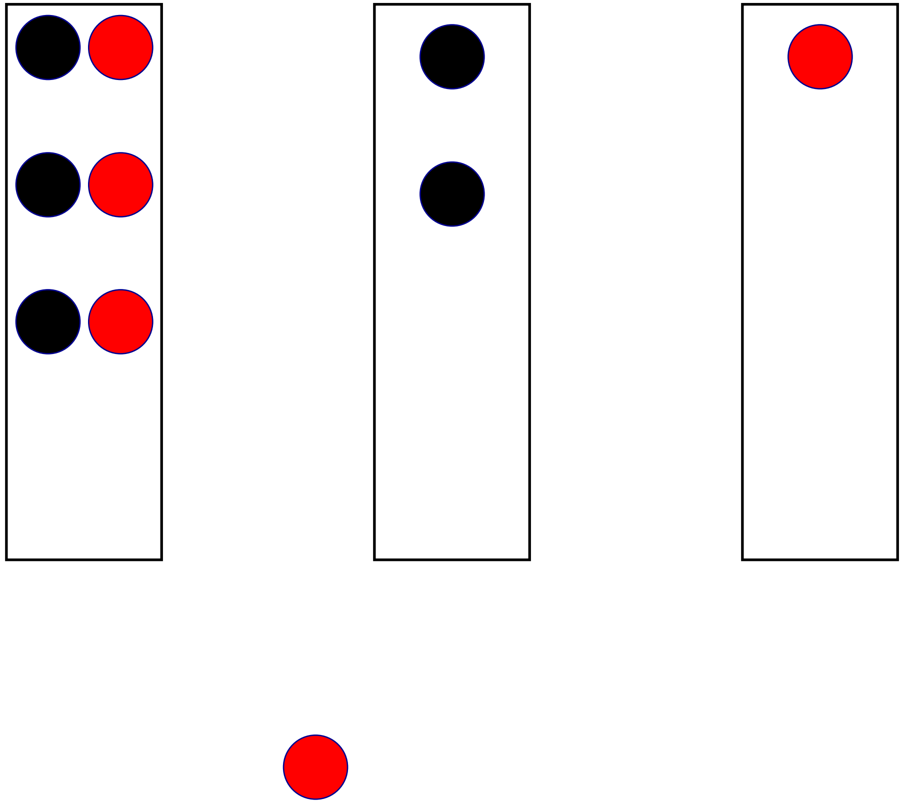
Unpaired Kidney Exchange

Z Unpaired donors ***D*** Unpaired patients ***P***



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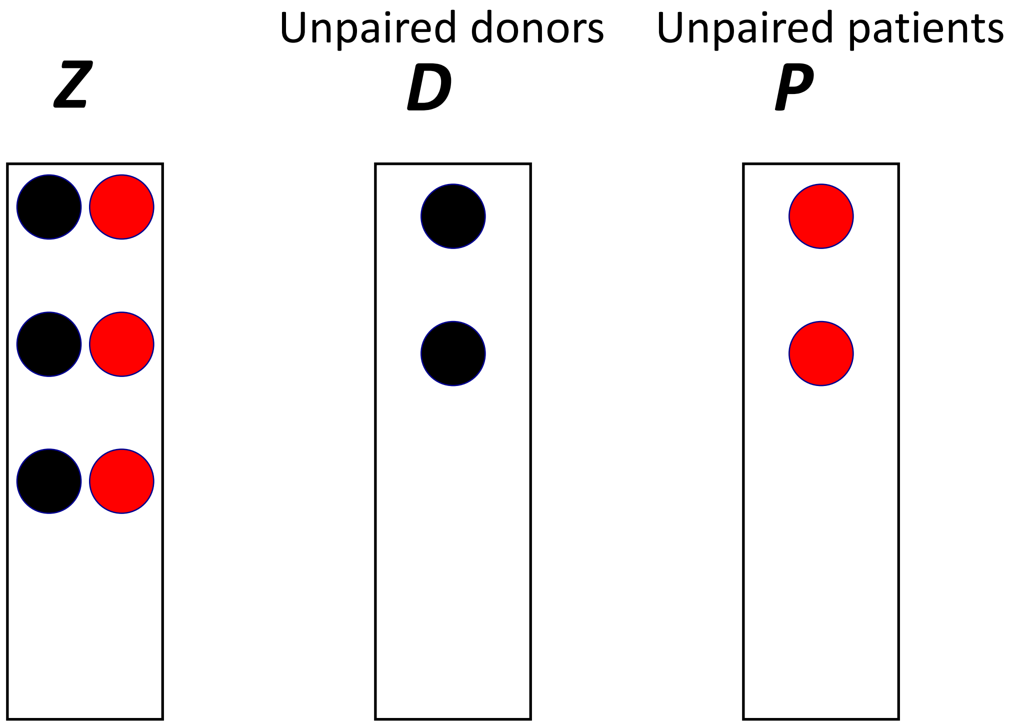

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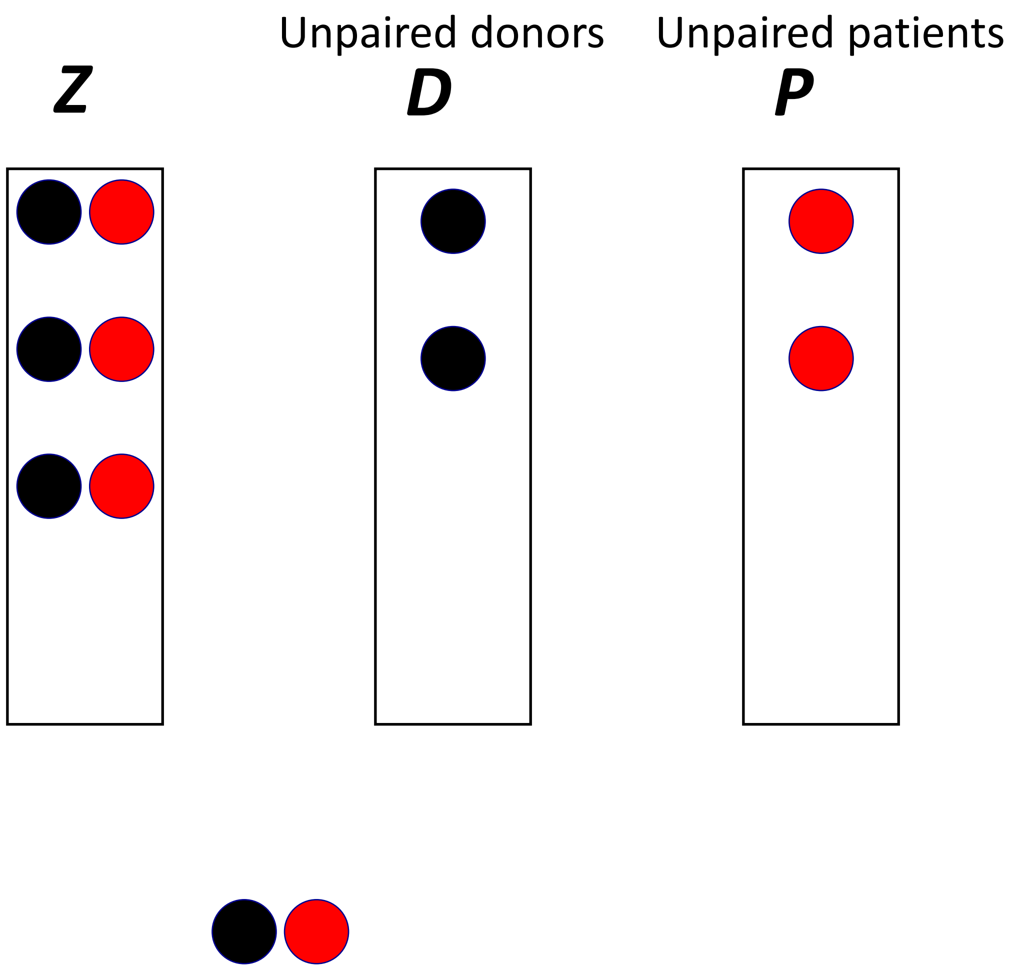

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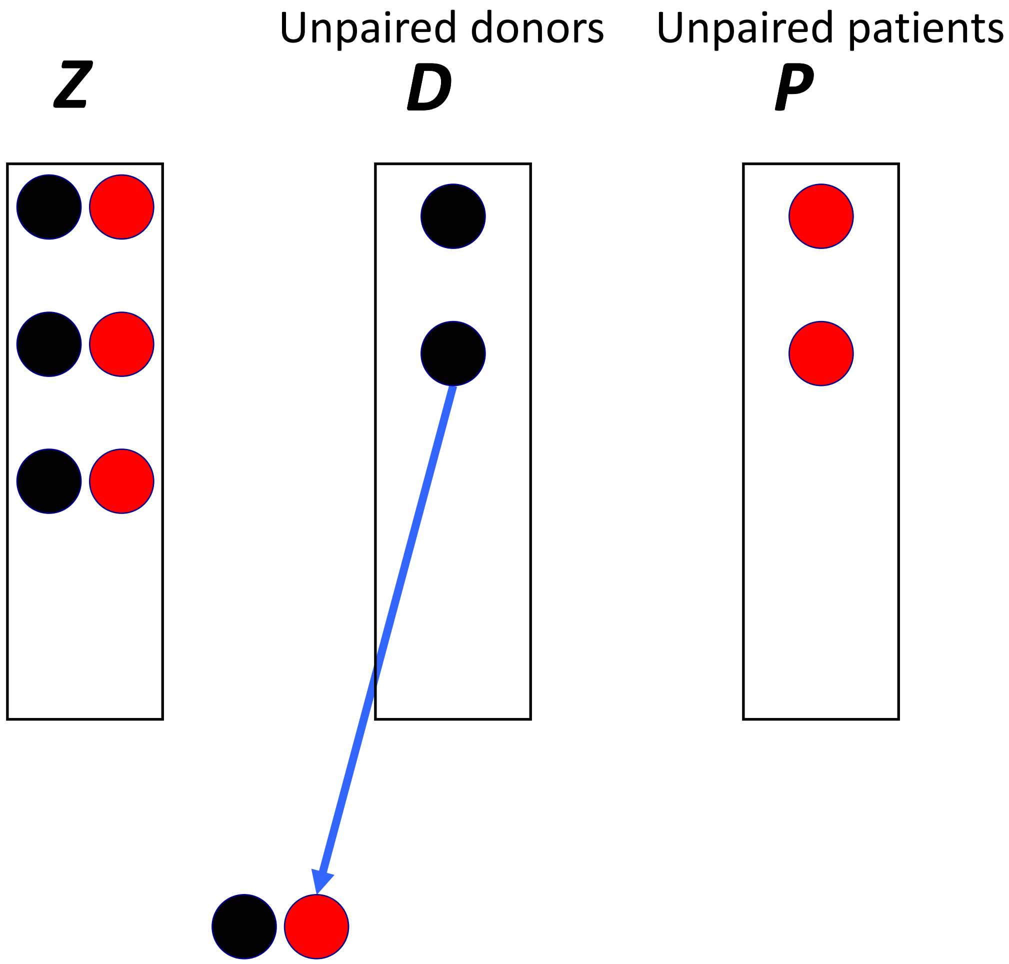

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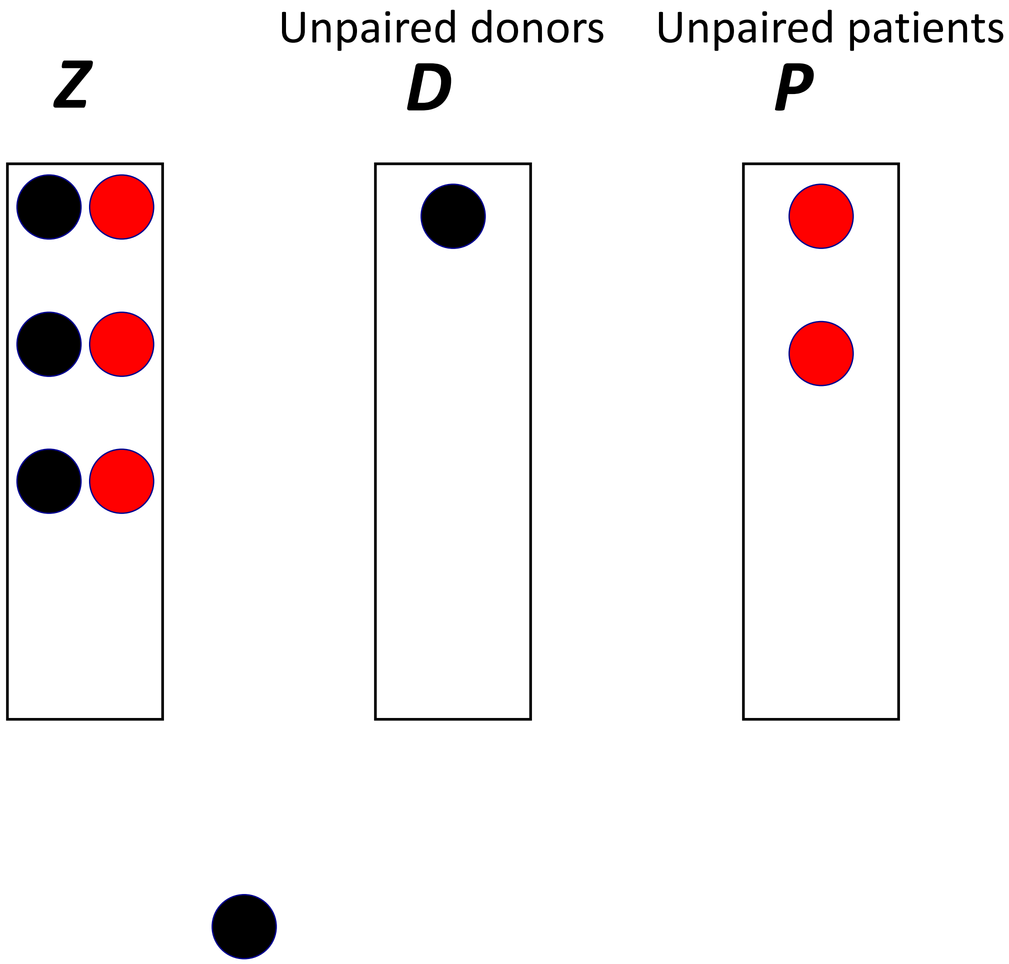

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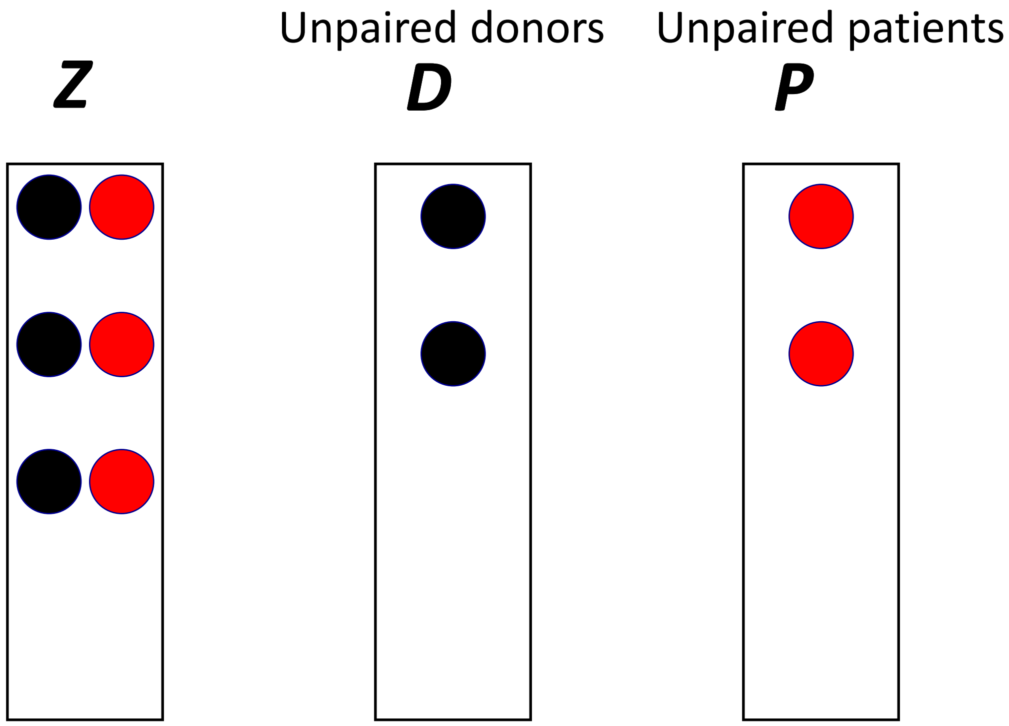

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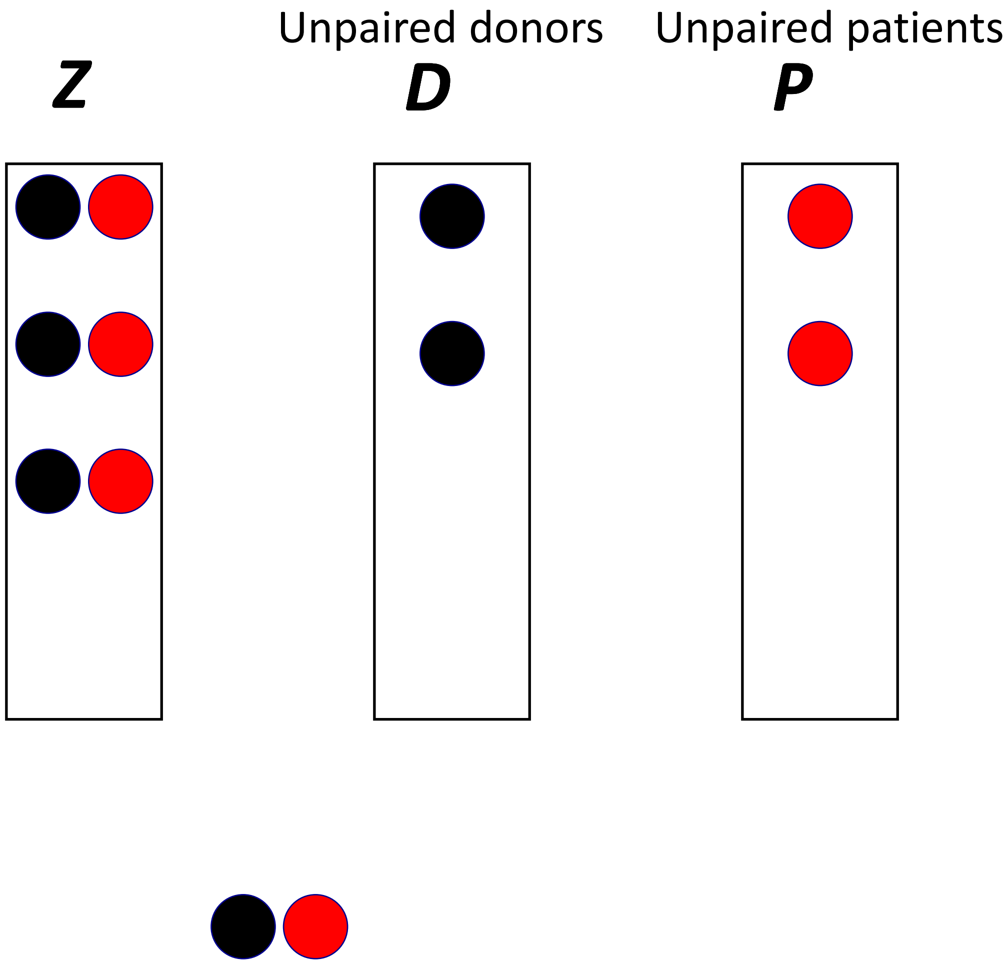

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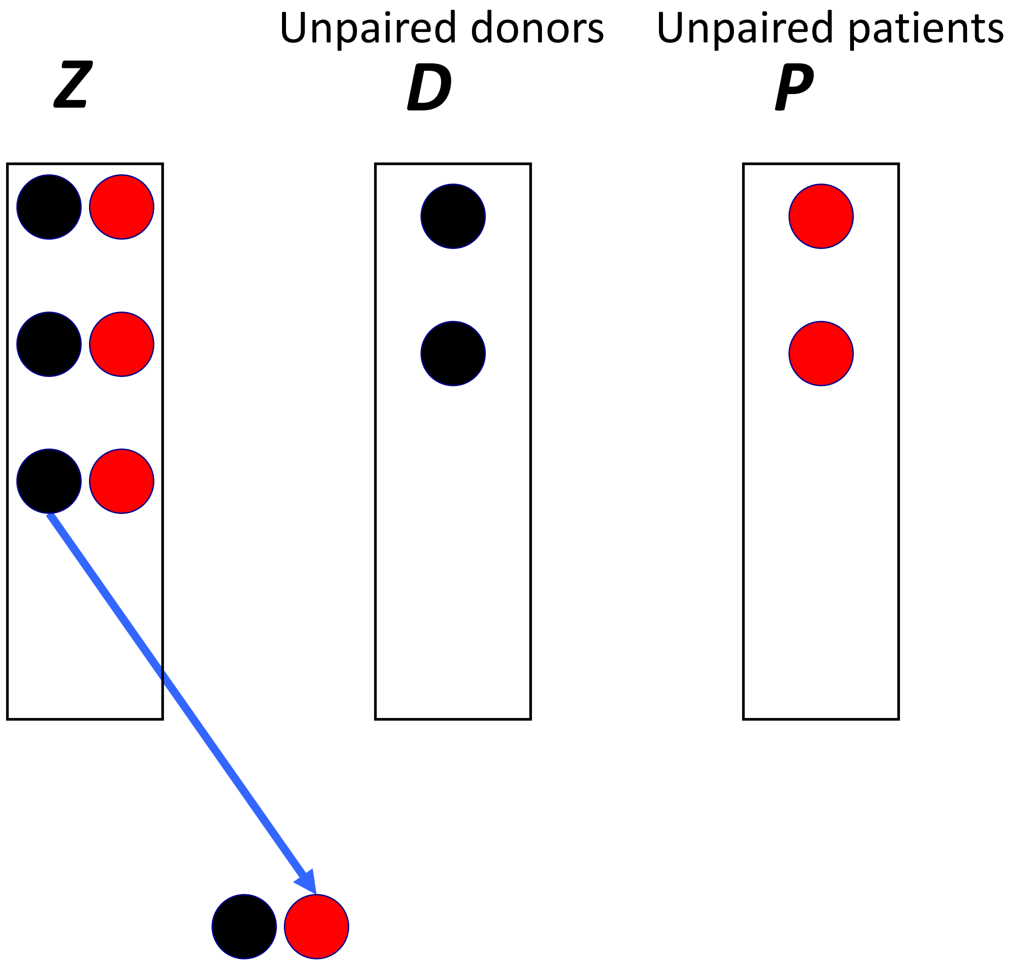

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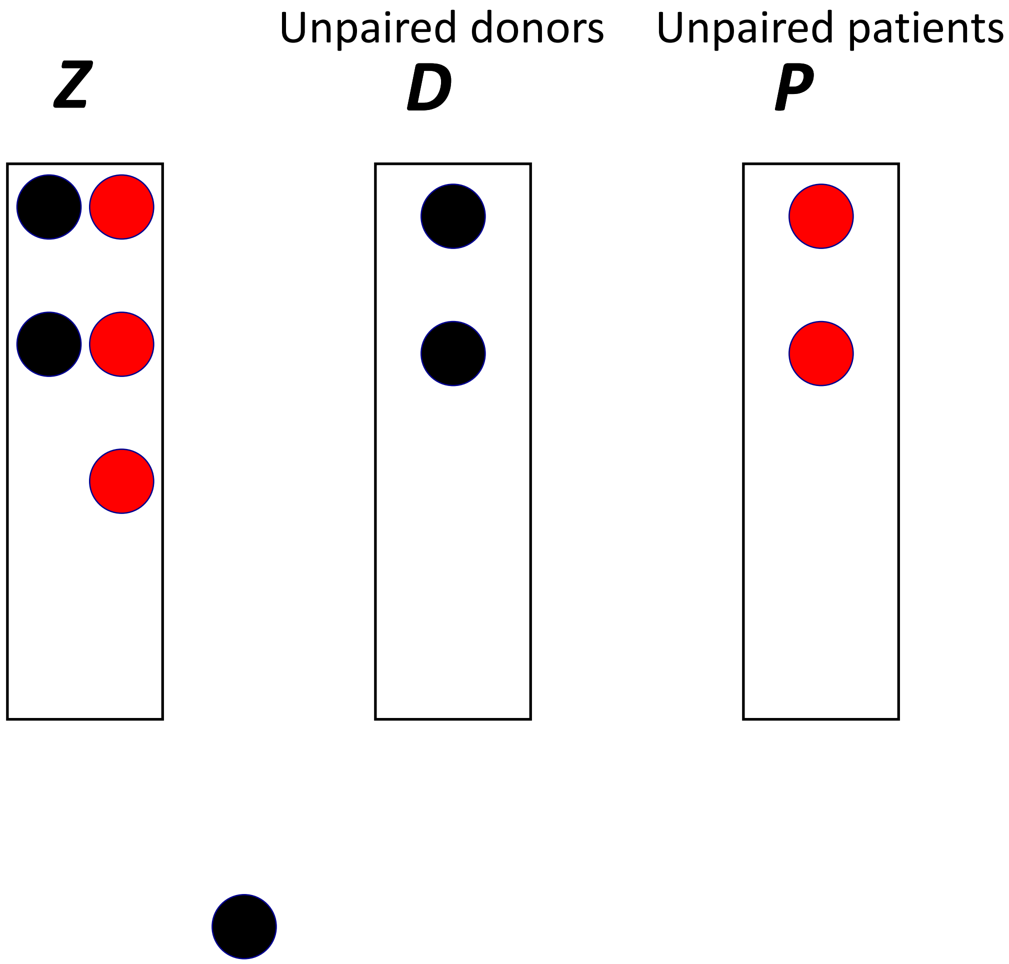

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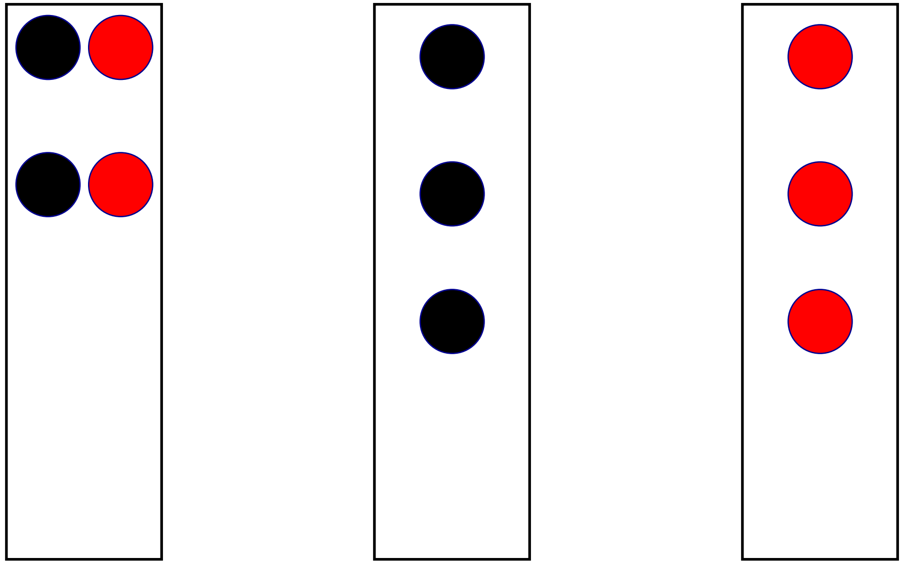
Unpaired Kidney Exchange

Z

Unpaired donors
D

Unpaired patients
P



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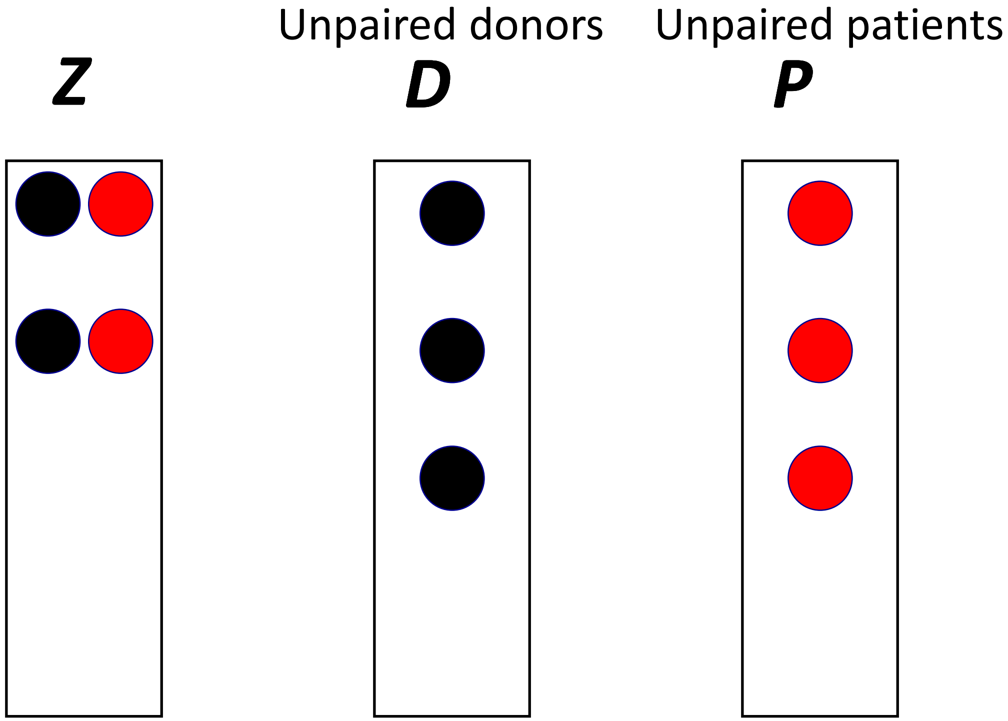

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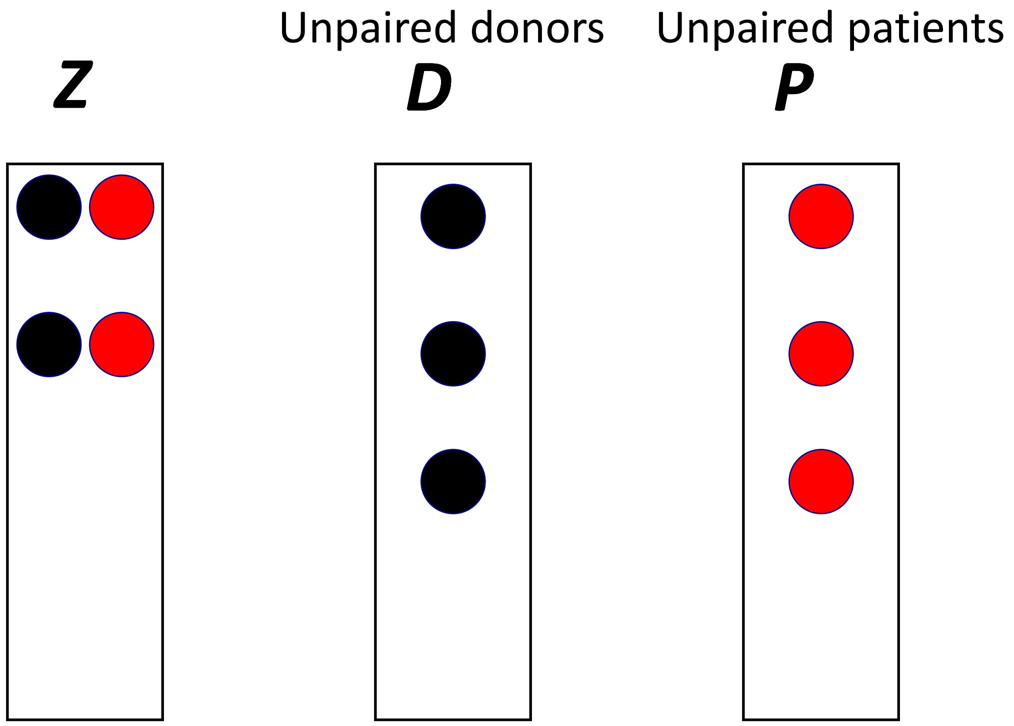

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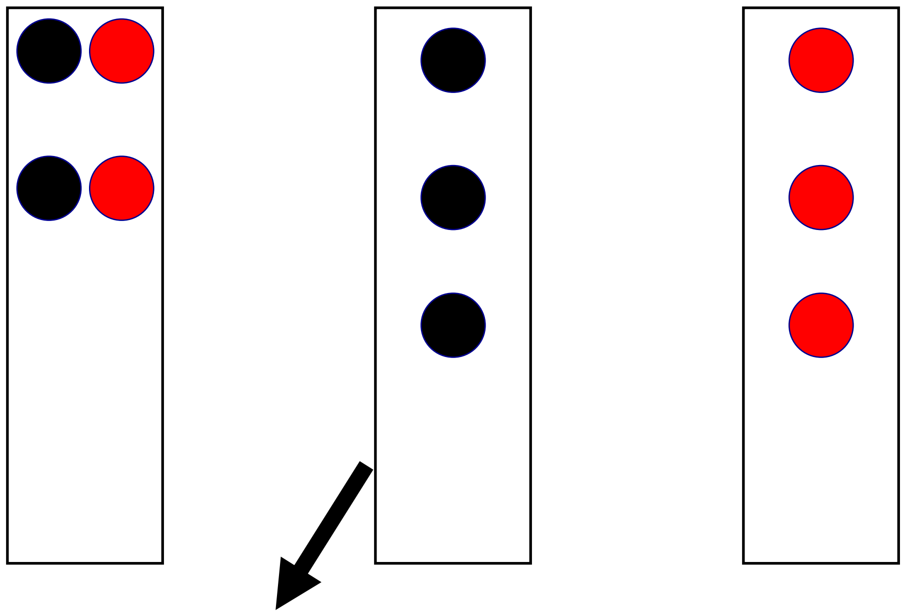
Two practical risks!

Unpaired Kidney Exchange

Unpaired donors Unpaired patients

Z *D* *P*



Renege risk for
unpaired donors in D
(same risk in Chains)

Two practical risks!

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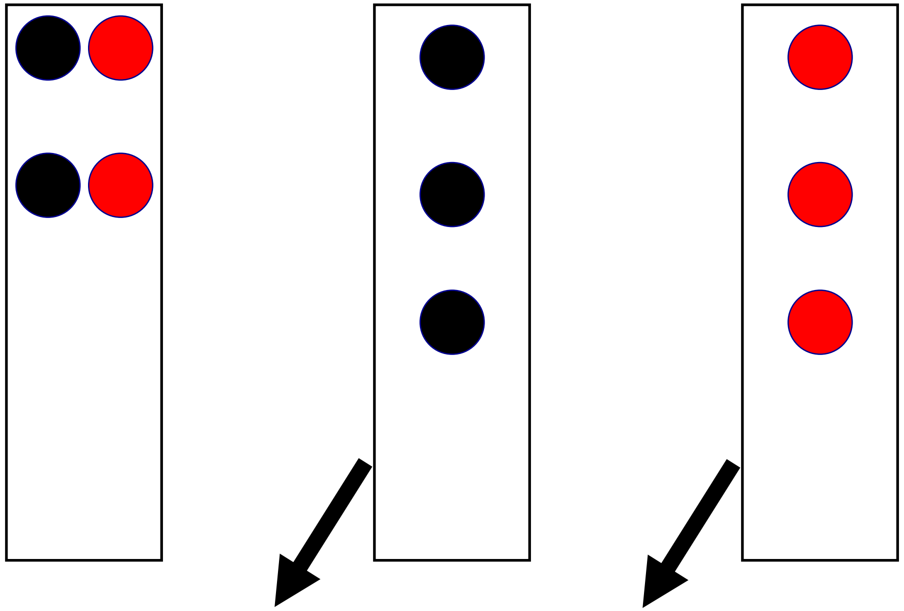
Unpaired Kidney Exchange

Unpaired donors　Unpaired patients

Z　***D***　***P***



Renege risk for unpaired donors in D (same risk in Chains)

Waiting time risk for unpaired patients in P

Two practical risks!

- ✓ **Donor gives after the patient receives**
- ✓ **Donor gives before the patient receives**

Preview

- **Unpaired matching algorithm**: A new mechanism to overcome double coincidence
  - A donor can donate even if her/his intended patient has not received yet: Waiting time risk
  - A patient can receive even if her/his intended donors has not given yet: Renege risk

  ⇒ **Realizes transplantations as soon as they are possible**

- **Theory**
  - A simple dynamic matching model: $\boldsymbol{W}\,(\mathit{Optimal}) \approx \boldsymbol{W}\,(\mathit{Unpaired}) < \boldsymbol{W}\,(\mathit{Chain}) < \boldsymbol{W}\,(\mathit{Pairwise})$

- **Empirics**
  - Counterfactual analysis on French kidney data
  - The two risks are weakened for large market simulations (FR, APD, NKR)
  - Modifying the algorithm with the help of deceased donors: **practical design**

  ⇒ Solves the two risks

Theory

Model (Ashlagi et al., 2019)

- Time is continuous

- Arrival a new patient-donor pair: Poisson process with rate $\boldsymbol{n}$

- Two possible types of patients: H(ard) to match $(\boldsymbol{\lambda})$ and E(asy) to match $(\mathbf{1}-\boldsymbol{\lambda})$

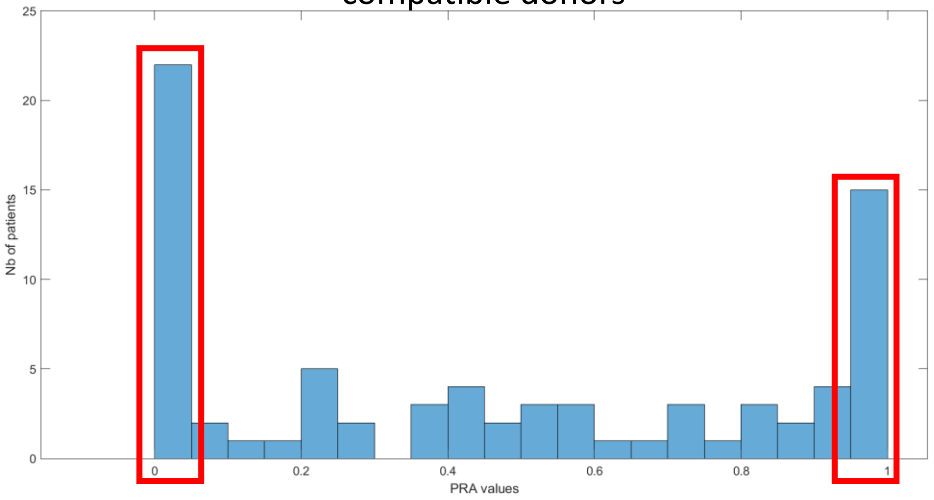
- **Hard-to-match (H)** patient is *compatible* with each donor w.p. $\boldsymbol{p_H}$ (independently of others).

- **Easy-to-match (E)** patient compatible w.p. $\boldsymbol{p_E} \leq 1$

- Patients and donors leave the market when they are matched (no exogenous exit)

Hard and Easy to match

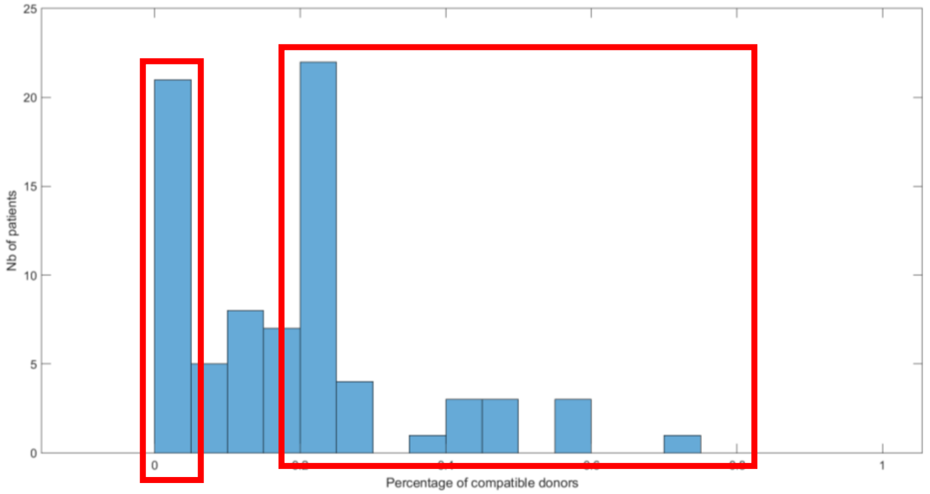
French patients percentage of HLA compatible donors among ABO compatible donors



Hard to match

Easy to match

French kidney patients percentage of compatible donors frequency



Hard to match

Easy to match

Objective

- Average waiting time at steady-state for **vanishingly small $\boldsymbol{p_H}$ and constant $\boldsymbol{p_E}$**

Definition: For a matching algorithm ALG, let

$$\boldsymbol{W}(ALG) = \mathrm{E}[\text{waiting time of patients}]$$

$$= \lambda W_H(ALG) + (1 - \lambda) W_E(ALG)$$

- We can show that for the algorithms we study $p_H W_E(ALG) \to 0$ as $p_H \to 0$

⇒ **Need to study the limit of $\boldsymbol{p_H W_H(ALG)}$**

- By Little's law, it is proportional to the steady state "pool size".

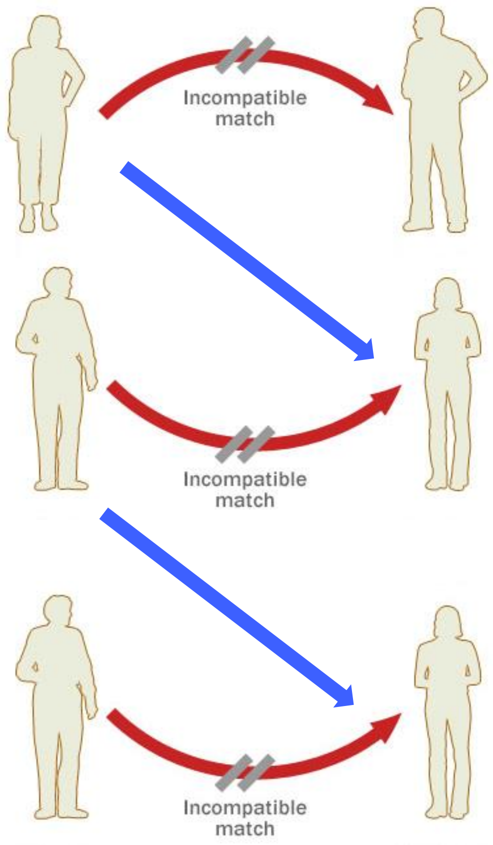
3 mechanisms to allocate kidneys

Pairwise: Match pairs upon arrivals to a compatible pair (if any), breaking ties arbitrarily.

Chain: At any date, there is a donor without a patient who is not compatible with any patient waiting. If he is compatible with the patient of a newly arrived pair: search ***randomly a maximal chain*** among the patients waiting starting with the donor of the newly arrived pair

Unpaired: Match patients upon arrival to a compatible donor (if any) and match donors upon arrival to a compatible patient (if any)..

How Chains Alleviate Double Coincidence?



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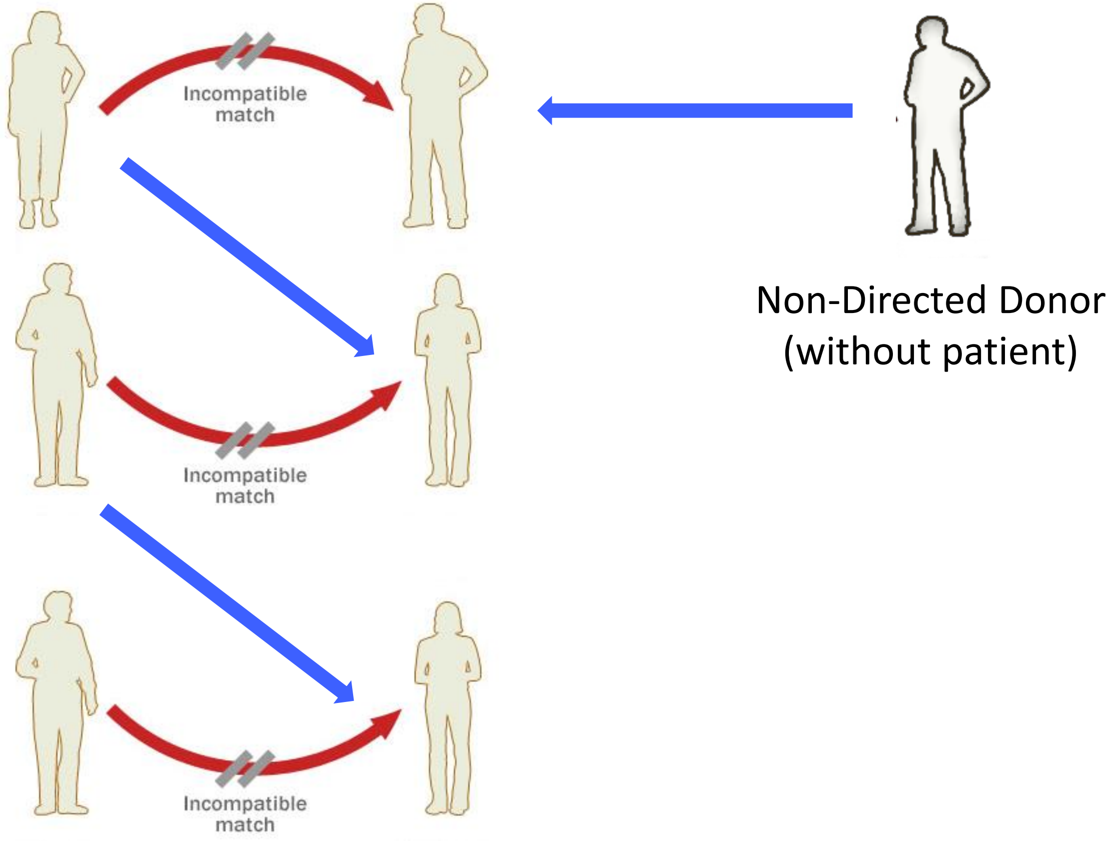

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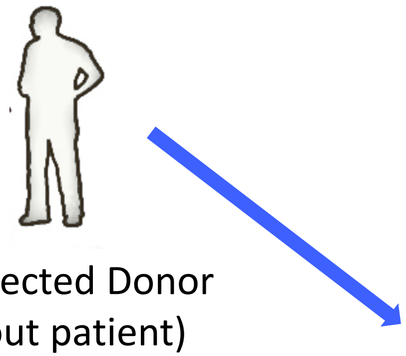
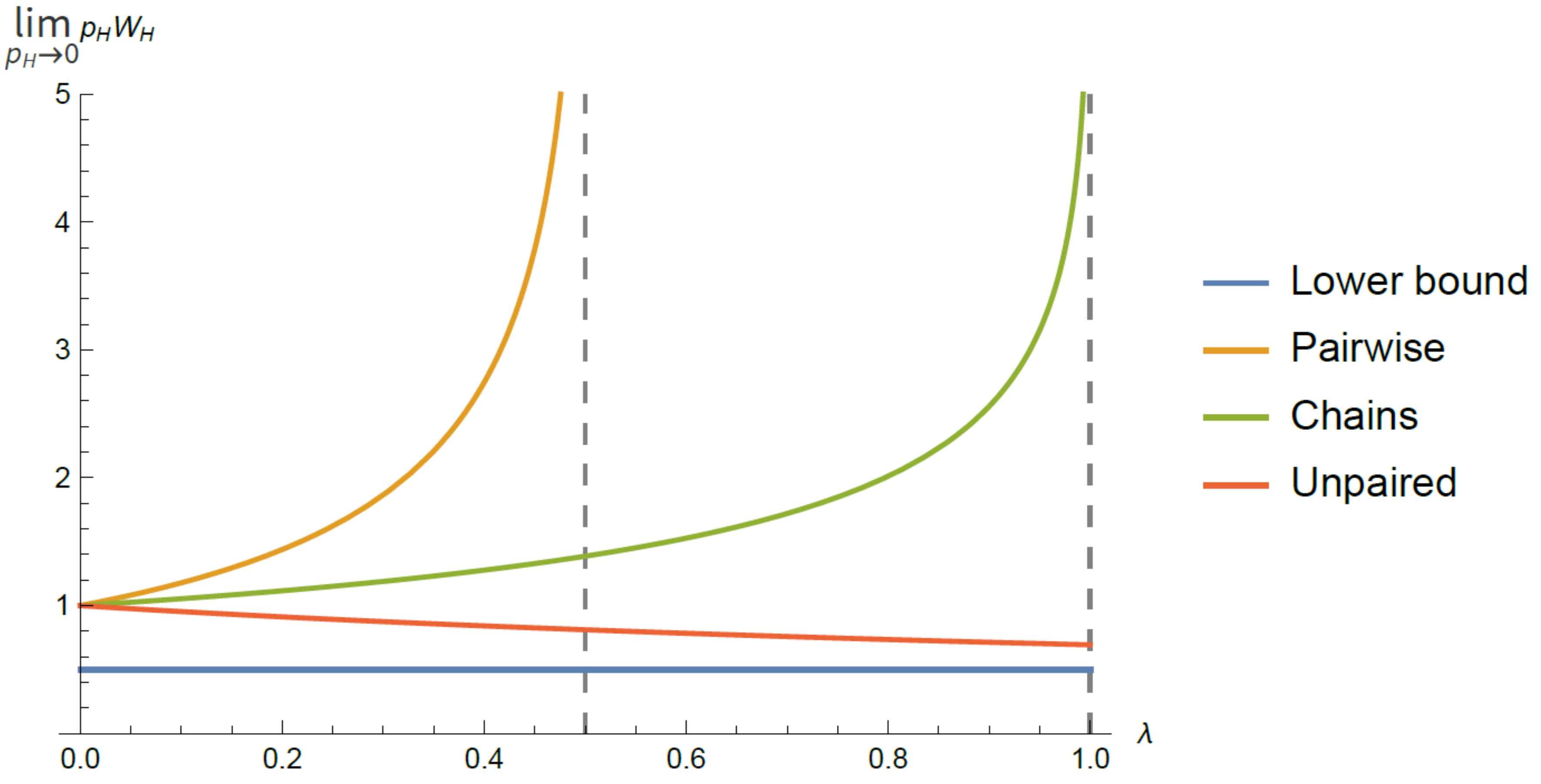


Results: Asymptotic waiting time of H patients

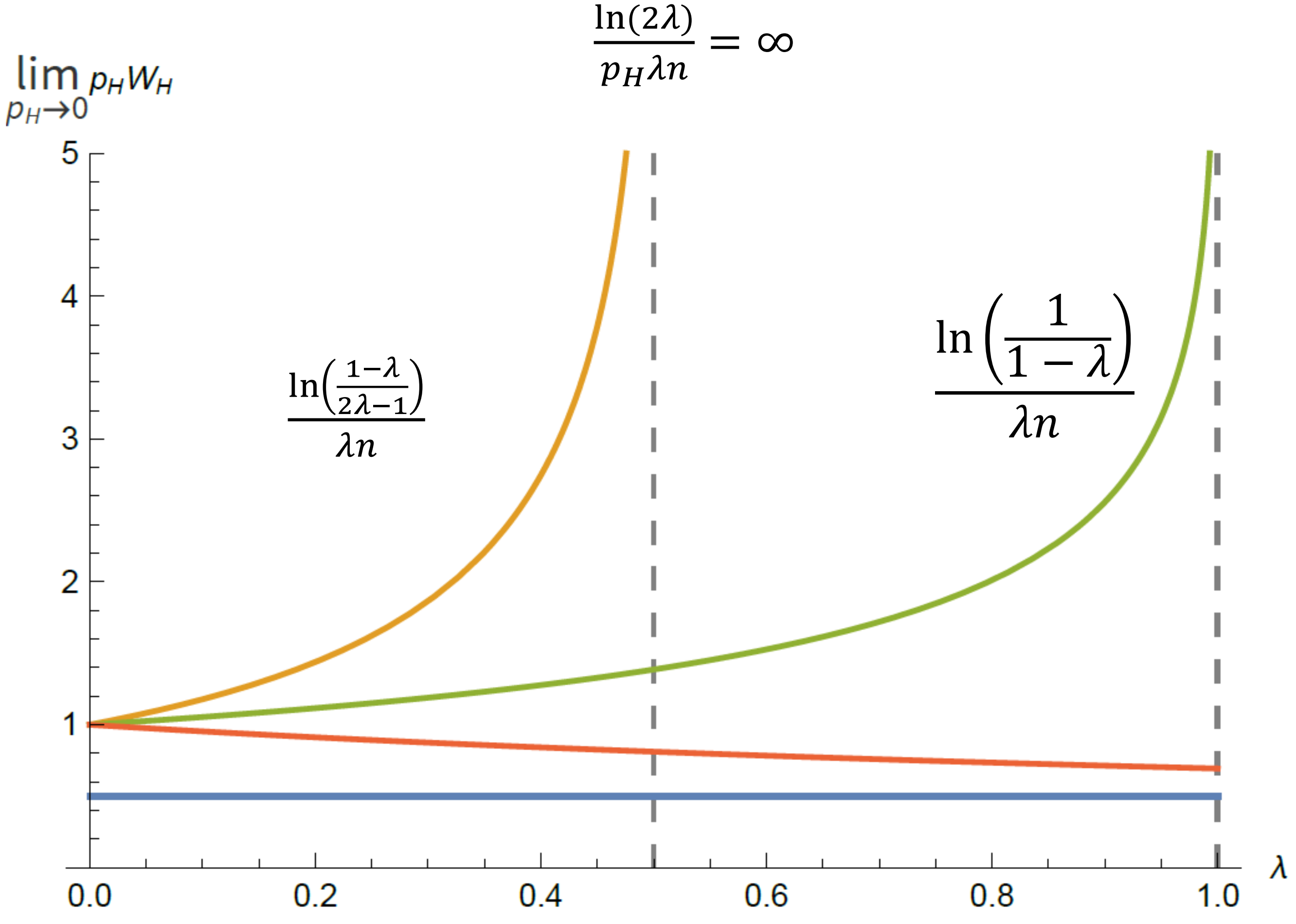
H patients

$$W\,(Optimal) \approx W\,(Unpaired) < W\,(Chain) < W\,(Pairwise)$$



Chain is better than Pairwise

H patients



Theorem 1 & Prop. 1 in Ashlagi et al. (2019)

$$W_H(Pairwise) = O\left(\frac{1}{p_H^2}\right) when\, \lambda \geq \frac{1}{2}$$

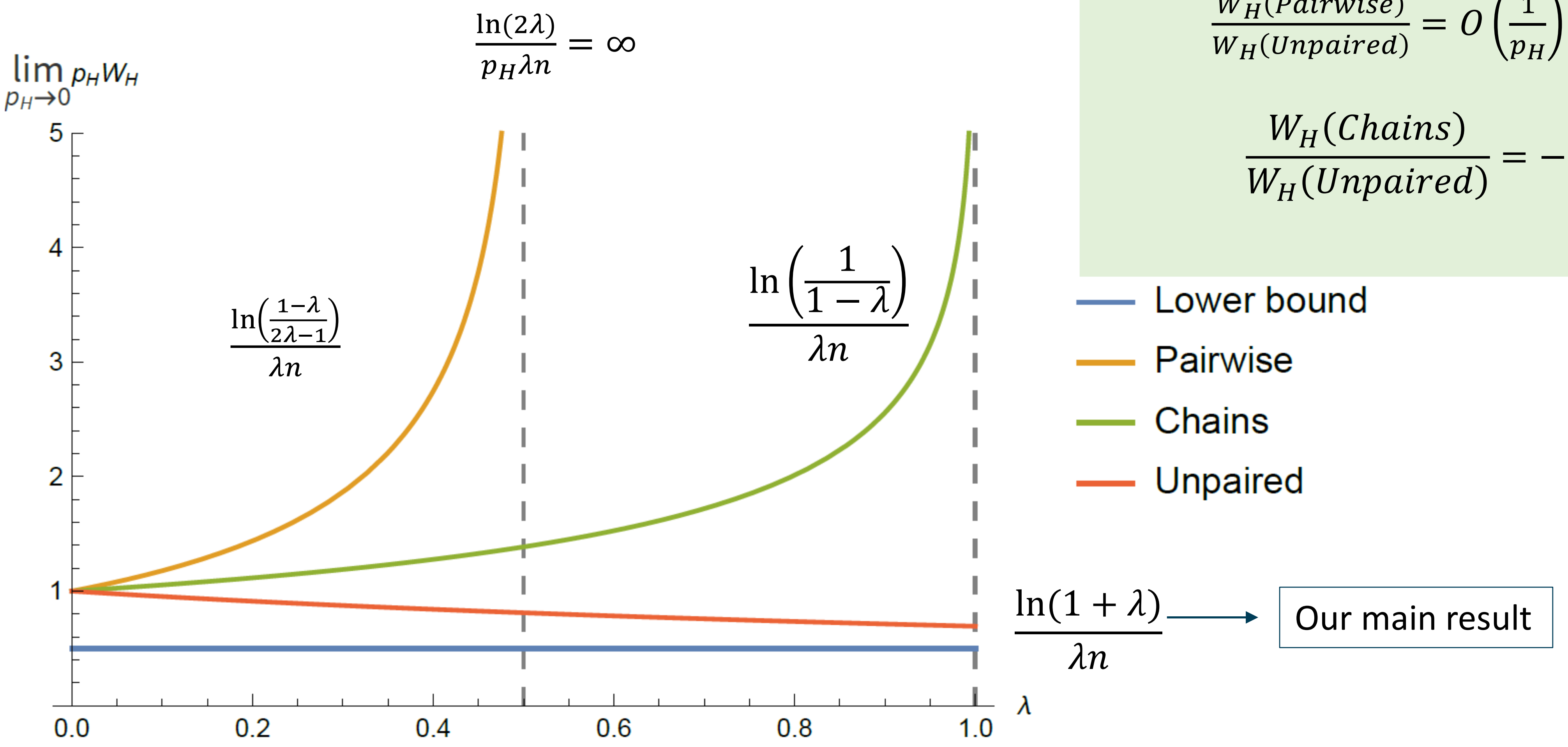
$$W_H(Chain) = O\left(\frac{1}{p_H}\right)$$

Unpaired improves upon Pairwise and Chain

$$\frac{W_H(Pairwise)}{W_H(Unpaired)} = O\left(\frac{1}{p_H}\right) \; when \, \lambda \geq \frac{1}{2}$$

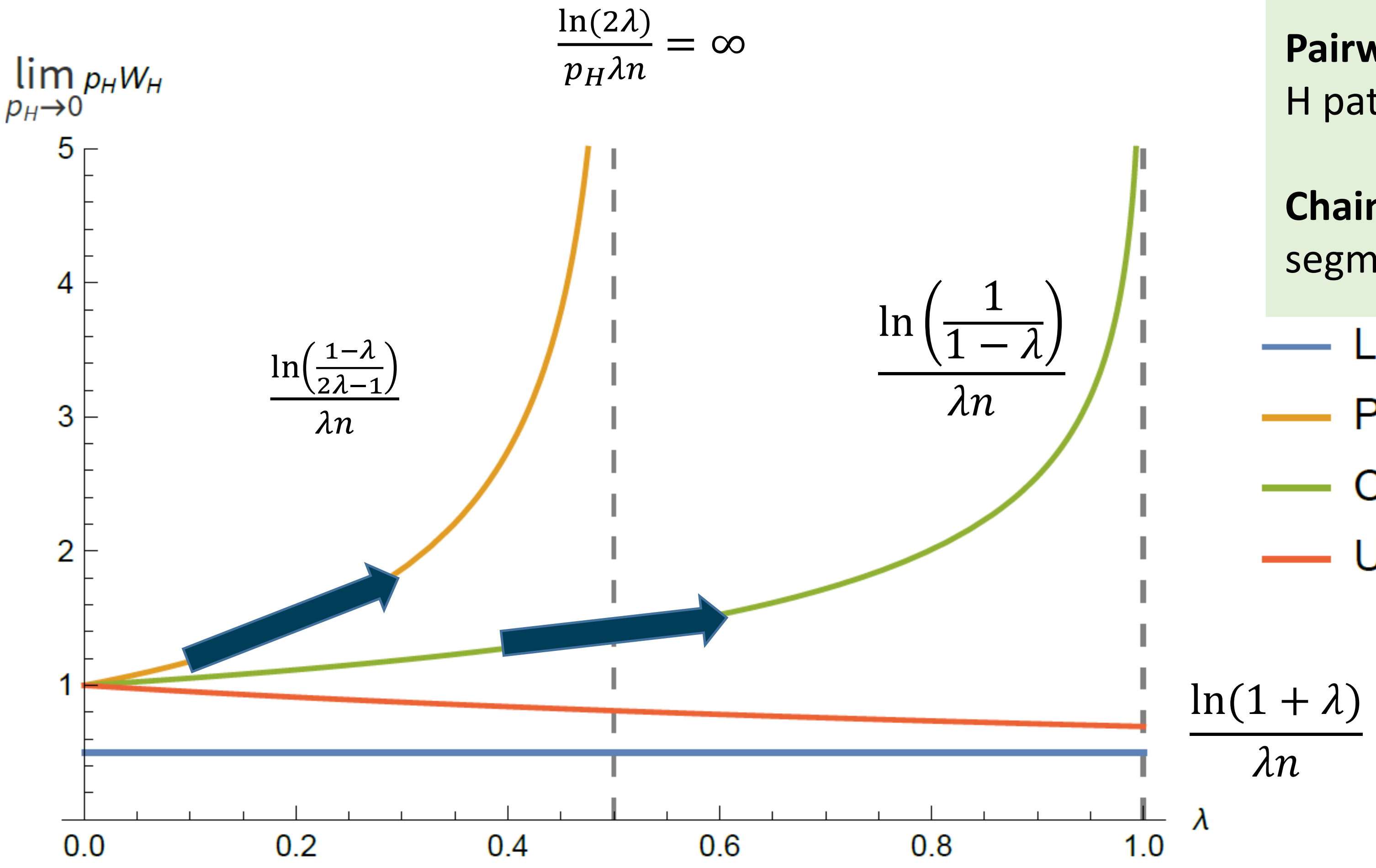
$$\frac{W_H(Chains)}{W_H(Unpaired)} = -\frac{\ln(1-\lambda)}{\ln(1+\lambda)}$$

H patients



Chain and Pairwise: the more E the better

H patients

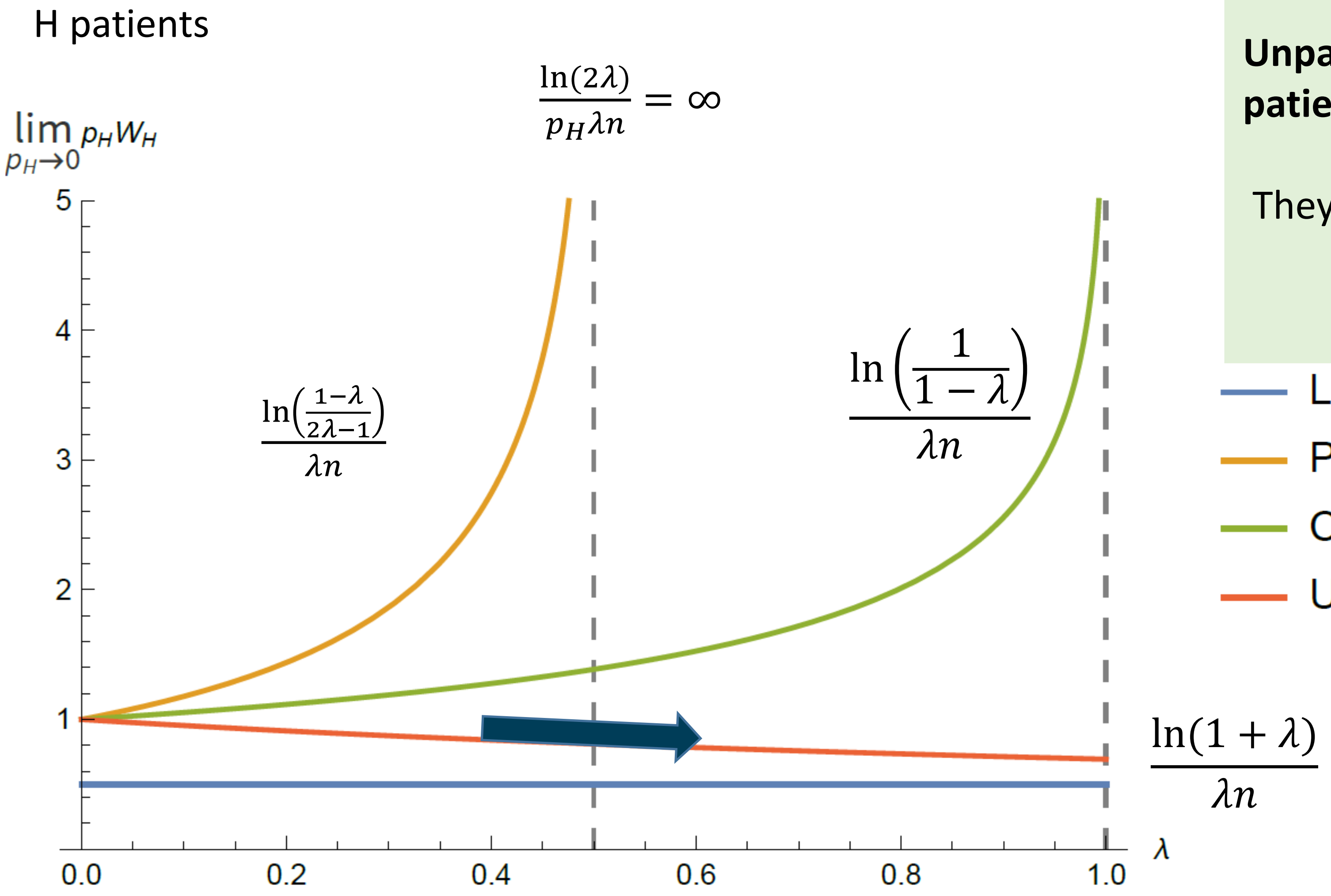


Need E patients to match H patients

Pairwise: E patients help for the cross compat. of H patients

Chain: E patients help to start more often new segments of the chains

Unpaired: the more E the worse for H

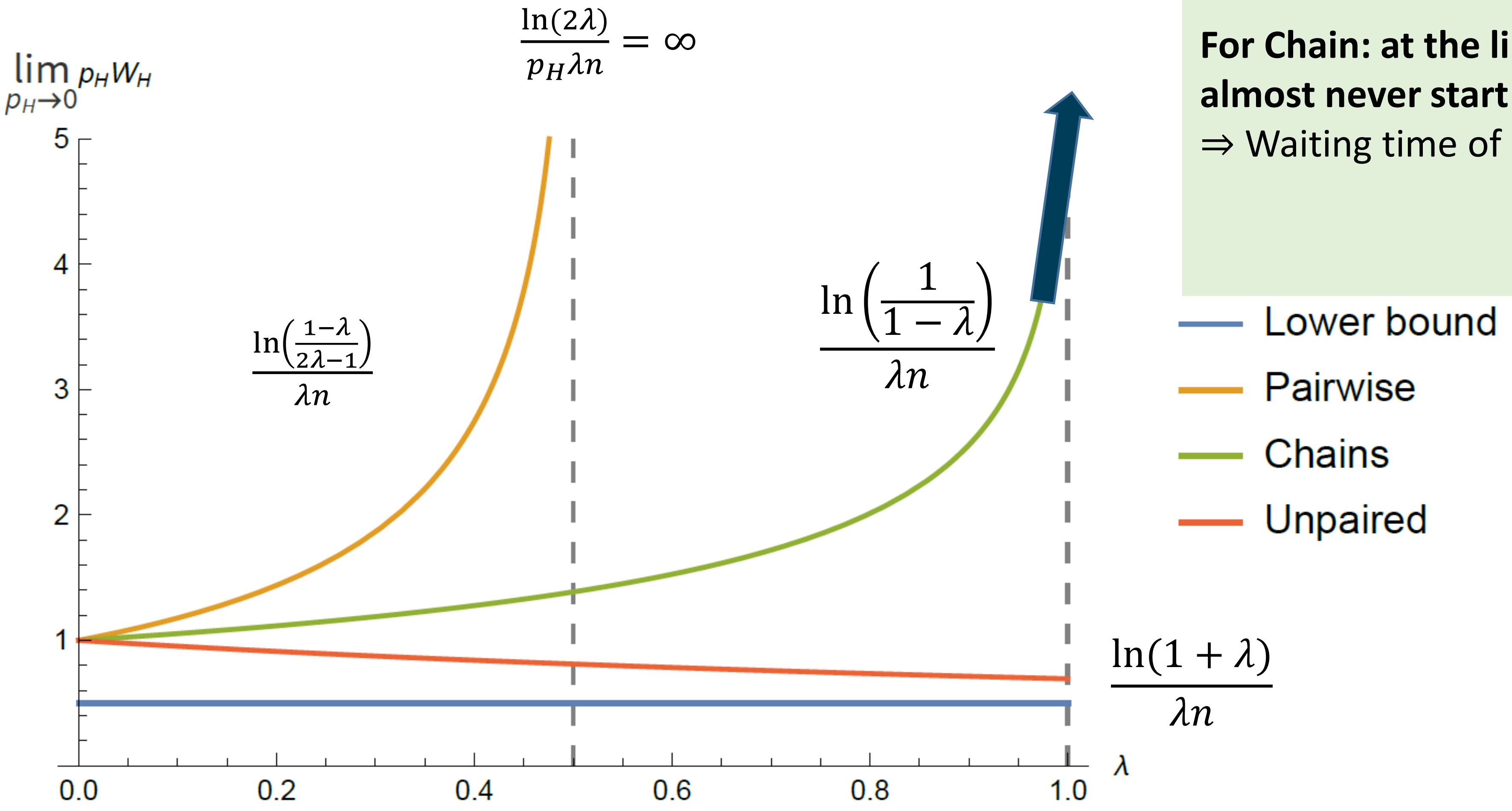


Unpaired does not need E patients to match H patients

They just take donors away from H patients

Chain is infinitely worse for H in the limit

H patients

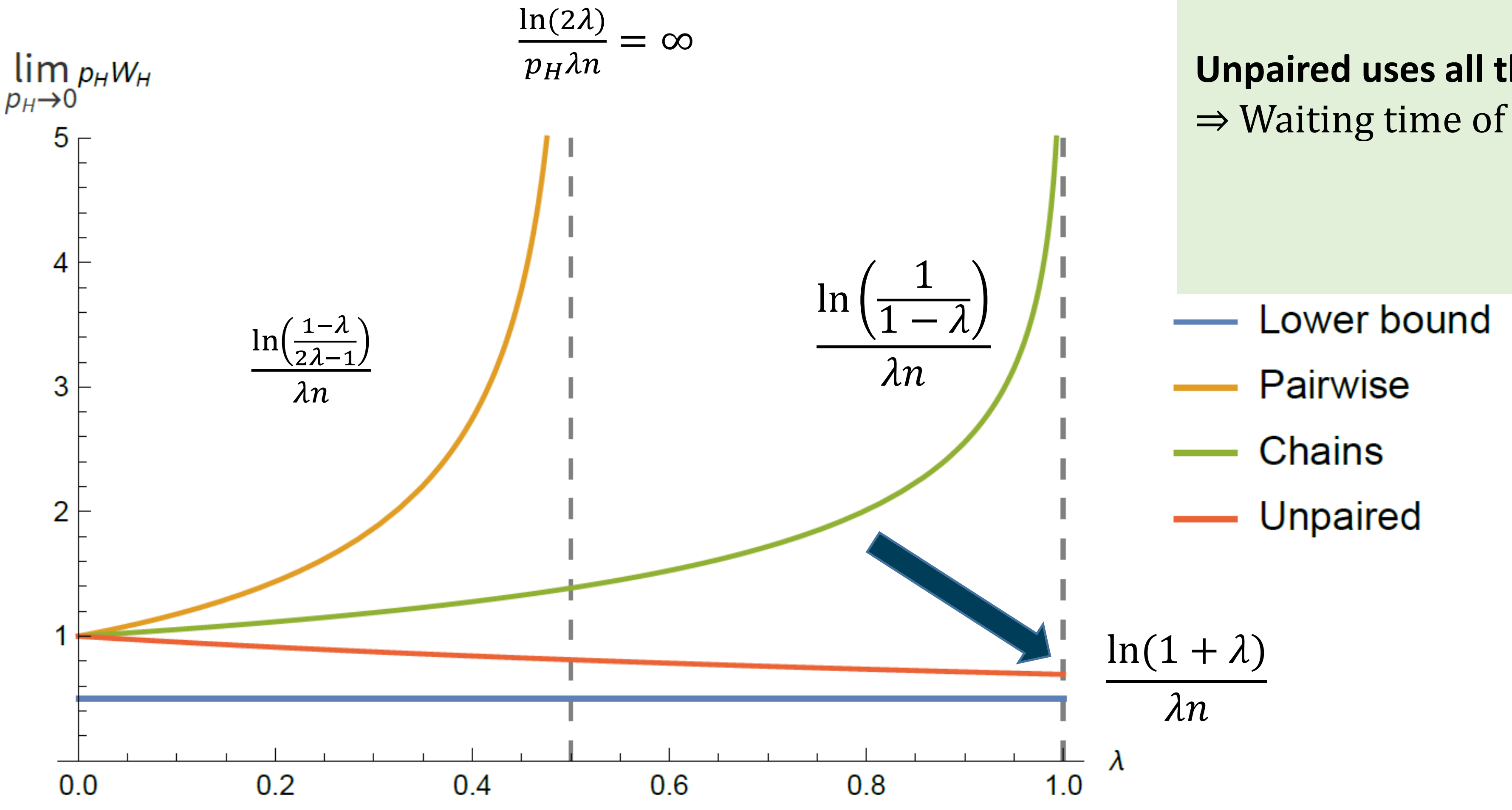


For Chain: at the limit, segment of the chain almost never start

⇒ Waiting time of H patients explodes

Unpaired is not infinitely worse in the limit

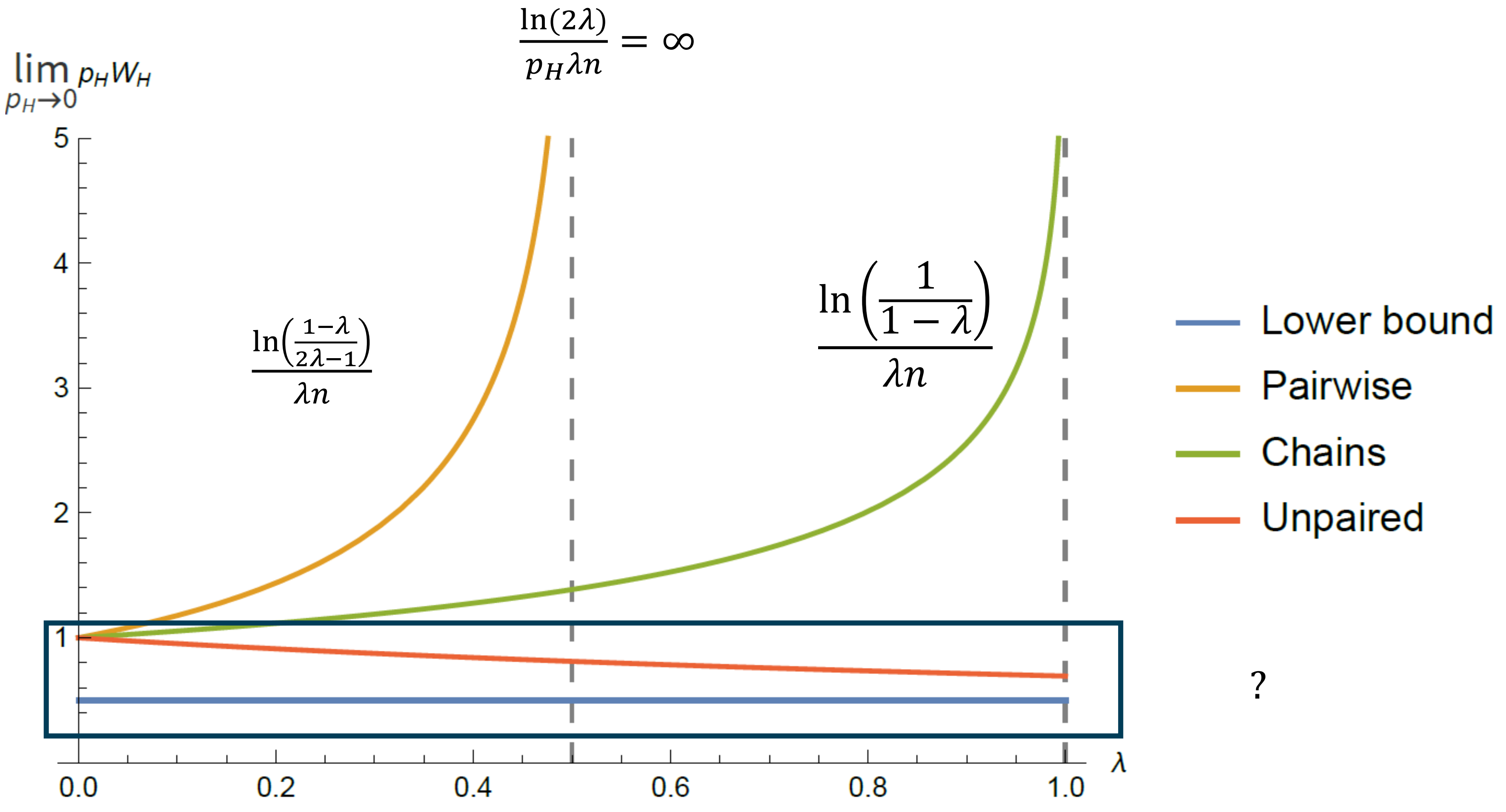
H patients



Unpaired uses all the donors available

⇒ Waiting time of H patients does not explode

What about other mechanisms?



The Optimal « Mechanism »

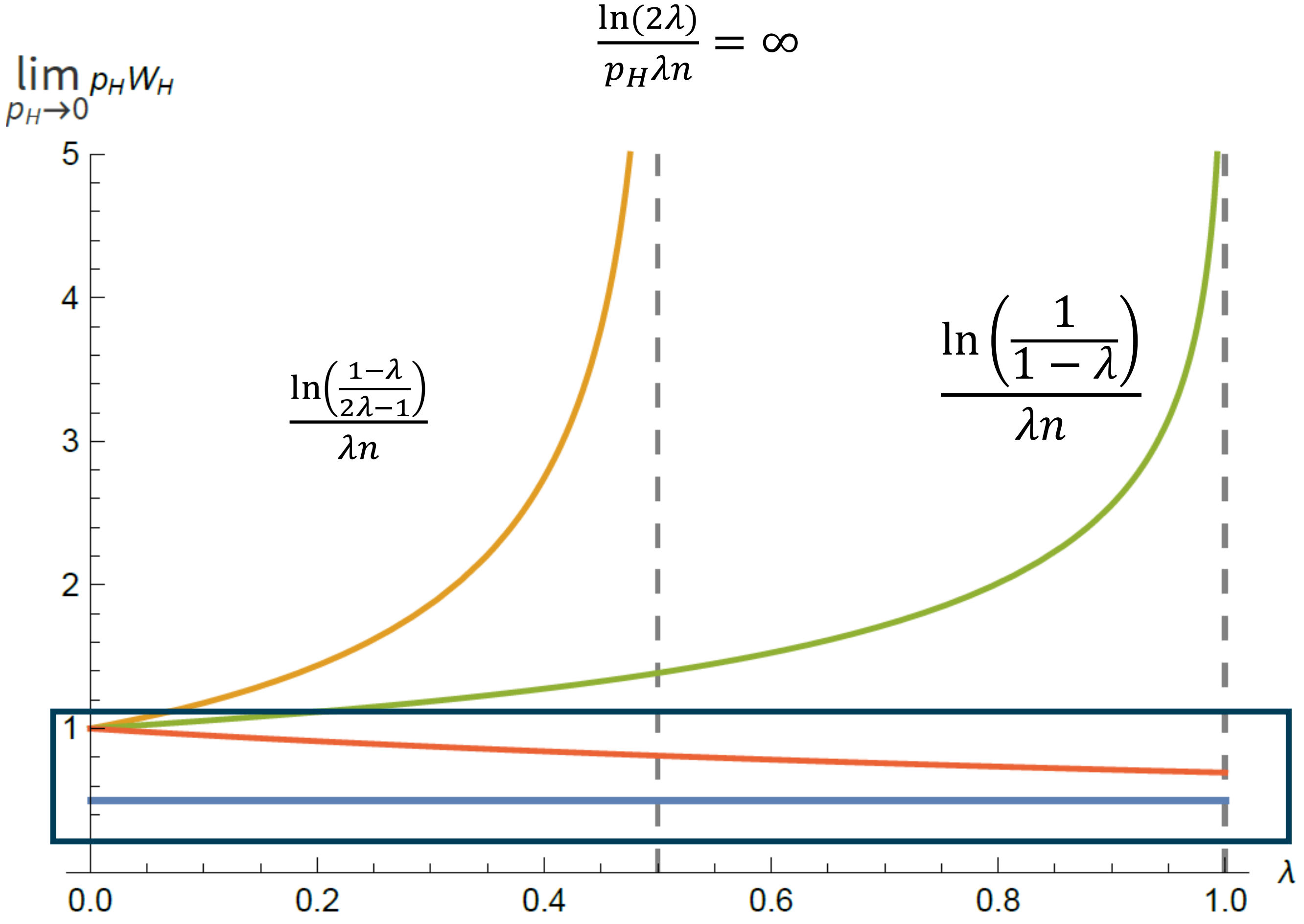
Defining the optimum is tricky, since an algorithm can avoid matching easy-to-match agents to match hard-to-match ones fast.

Definition: The **Optimal Matching Algorithm (OPT)** solves:

$$\boldsymbol{W(OPT)} := \boldsymbol{Inf} \quad \mathbf{W}(\mathrm{ALG})$$

s.t. $\boldsymbol{ALG}$ induces a stationary distribution

Unpaired cannot do worse than 2 times the W of any other mechanism



In steady-state:

$$\frac{W(Unpaired)}{W(OPT)} \leq 2$$

$ratio \ \leq 2$

To wrap up

$$W\,(Optimal) \approx W\,(Unpaired) < W\,(Chain) < W\,(Pairwise)$$

- ✓ Chain is bounded away from the Optimal
- ✓ Unpaired closes the gap: even if it is a simple greedy algorithm!

Empirics

Data: French KEP + DDL

- KEP started in December 2013: **only pairwise exchanges are allowed**
- Centralized at the national level

- From Dec. 2013 to Feb. 2018: **78 pairs participated to the KEP**
  - ✓ Average pool size: **18 pairs per match-run**
  - ✓ Patients grafted: 15% (12 transplants)

- Competing method: "desensitization" **508 incompatible grafts** during that same period
- Data on true compatibility + all the DDL kidneys (date, compat., charact…)

⇒ **We draw randomly arrival times consistent with participation + no exit**

Results: Baseline on the French KEP

	Pairwise	Chain (+ Pairwise)	Unpaired	Omniscient (best ex post)
Avg % of grafts	29%			
Avg Waiting time (days)	731			
Med. Waiting time in P (days)	-			
Med. Waiting time in D (days)	-			

Results: Baseline on the French KEP

	Pairwise	Chain (+ Pairwise)	Unpaired	Omniscient (best ex post)
Avg % of grafts	29%	30%		
Avg Waiting time (days)	731	701		
Med. Waiting time in P (days)	-	-		
Med. Waiting time in D (days)	-	83		

Results: Baseline on the French KEP

	Pairwise	**Chain (+ Pairwise)**	**Unpaired**	**Omniscient (best ex post)**
Avg % of grafts	29%	30%	57%	
Avg Waiting time (days)	731	701	440	
Med. Waiting time in P (days)	-	-	220	
Med. Waiting time in D (days)	-	83	305	

Results: Baseline on the French KEP

	Pairwise	Chain (+ Pairwise)	Unpaired	Omniscient (best ex post)
Avg % of grafts	29%	30%	57%	58%
Avg Waiting time (days)	731	701	440	425
Med. Waiting time in P (days)	-	-	220	474
Med. Waiting time in D (days)	-	83	305	475

Results: Baseline on the French KEP

	Pairwise	Chain (+ Pairwise)	Unpaired	Omniscient (best ex post)
Avg % of grafts	29%	30%	57%	58%
Avg Waiting time (days)	731	701	440	425
Med. Waiting time in P (days)	-	-	220	474
Med. Waiting time in D (days)	-	83	305	475

Match rate of unpaired greedy similar to omniscient but…

… **the two risks: renege + waiting time in P can be problematic.**

How to reduce the two risks?

- We can integrate Unpaired together with the Deceased Donor List (DDL)
  - ➢ **Waiting time risk:** offer **good quality** deceased donors to unpaired patients in P
  - ➢ **Renege risk:** make unpaired donors in D give "quickly" to patients on the DDL
    - When using a DD kidney for an unpaired patient in P or as soon as his paired patient receives

- Small market issue?

⇒ We simulate larger markets (FR, APKD, NKR)

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- Small market issue?

⇒ We simulate larger markets (FR, APKD, NKR)

Strongly weakens the two risks

How to reduce the two risks?

- Use a DD kidney for an unpaired patient in P only if an unpaired donor in D gives to a DDL patient (select only good quality DD based on KDPI)

	Pairwise	Chain (+ Pairwise)	Unpaired	Unpaired With DDL
Avg % of grafts	29%	30%	57%	82%
Avg Waiting time (days)	731	701	440	190
Med. Waiting time in P (days)	-	-	220	7
Med. Waiting time in D (days)	-	83	305	38

The risks are weakened (median time in P is 7 days!)

Ongoing new results

- The pool is very small: scalability of the results with DDL ?
  - ↗ participation ⇒ ↗ living donations ⇒ ↘ waiting time in P + ↘ use of the DDL
  - Our current large market simulations confirm it: more refinements needed

- Link between the theory without DDL and the empirical analysis
  - Bound with the Optimal waiting time can change
  - Verify the claim that more pairs participate ⇒ Use less the DDL

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- Link between the theory without DDL and the empirical analysis

⇒ **A simple variation of the model has a waiting time divided by 2**

  - Bound with the Optimal waiting time can change

  ⇒ **The bound with a sligthly modified definition of Optimal is similar**

  - Verify the claim that more pairs participate ⇒ Use less the DDL

  ⇒ **Ongoing using another simple variation of the model: we conjecture it is true**

Large markets

- 2 types of simulations

1. French KEP: includes all the pairs who did « desensitization » (+508)

⇒ Allows us to simulate Unpaired with DDL

2. Simulation of U.S. KEPs: APKD and NKR using the method of Ashlagi et al. (2019) and calibrations from Ashlagi and Roth (2020)

⇒ Simulate bigger pools (but simulated data so no DDL)

Large markets: FR with desensitized pairs

	French KEP + Desensit pairs			
	Pairwise	Unpaired	Unpaired with DDL	Omn.
Size	586	586	586	586
% grafts	44%	67%	88%	69%
Waiting Time	470	269	111	254
Waiting time in P	-	176	5	311
Waiting time in D	-	196	34	305

Large markets: U.S. simulations

	APKD			NKR		
	Pairwise	**Unpaired**	**Omn.**	**Pairwise**	**Unpaired**	**Omn.**
Size	804	804	804	2390	2390	2390
% grafts	47%	68%	70%	56%	73%	74%
Waiting Time	479	284	265	392	237	222
Waiting time in P	-	203	447	-	102	431
Waiting time in D	-	182	346	-	90	252

Conclusion

- **In theory:** $\boldsymbol{W\,(Optimal) \approx W\,(Unpaired) < W\,(Chain) < W\,(Pairwise)}$

- **Empirically:**

  ✓Theoretical results are confirmed

  ✓The two issues are weakened when using DDL + in large markets

  ✓ Final version can match up to 80% of the market

Conclusion

- **Ongoing**

  - Simple variations of the model to include the DDL
  - More empirical results to support the scalability when using the DDL
  - Precise final proposal for Unpaired and simulate it
    - Use of DDL and choice of patients to be unpaired and join P
    - Ask donors to give “quickly” to DDL patients to avoid renege