Regulating Product Communication*

Maarten C.W. Janssen[†]
University of Vienna,
National Research University Higher School of Economics and CEPR

Santanu Roy[‡]
Southern Methodist University

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Abstract

Information regulation that penalizes deceptive communication by firms can have significant unintended consequences. We consider a market where competing firms communicate private information about product quality through a combination of pricing and direct communication (such as advertising or labeling) and where quality claims communicated directly may be deliberately false. A higher fine for lying reduces the reliance on price signaling, thereby lowering market power and consumption distortions; however, it may create competitive incentives for excessive disclosure. Low fines are always worse than no fines. High fines are welfare improving only if (truthful) disclosure itself is rather inexpensive. Generally, high and low quality firms may both worse off under fines on false claims.

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†Department of Economics, University of Vienna, Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria. E-mail: maarten.janssen@univie.ac.at

‡Department of Economics, Southern Methodist University, 3300 Dyer Street, Dallas, TX 75275-0496. E-mail: sroy@smu.edu

1 Introduction

For a large class of goods ranging from financial products (such as mortgages and bonds) to consumer durables and electronics, it is difficult for potential buyers to directly observe all product quality attributes. The same is true for a wide class of services such as health and educational services as well as many B2B markets. Quality attributes of interest to buyers include ethical dimensions of the production process as environmental footprint, the working conditions of employees (including the use of child labor), employment discrimination, the use of genetically modified organisms and prices paid to suppliers (fair trade). In most of these markets, sellers are likely to have more information about the quality attributes of interest than buyers.[1] Sellers of products with more favorable quality attributes may want to communicate this kind of private information directly to buyers to increase the demand for their own products.

There are several reasons why, in many industries, communicating convincing information about product attributes to buyers is significantly costly. First, communication channels that can easily reach most buyers, such as mass media advertising and direct marketing, are often quite costly to use. Second, some product attributes (for example, technical specifications) can often be complex and communicating information about these attributes in a commonly understandable form requires significant investment in information production and dissemination. Finally, certain modes of communication such as third party certification, or ratings by a reputed agency or public authority that are more credible to buyers (than say, Twitter or Facebook postings) are also significantly costly.

However, simply using a costly channel to communicate product information does not necessarily make it credible to buyers.[2] In the aftermath of the last financial crisis, there were concerns that as the rating agencies are paid by the financial institutions, ratings may not reflect true quality. Similar concerns have been expressed for certain environmental and other ethical product labels. By making it more costly for firms to lie or misrepresent their product quality attributes, public policy can increase the credibility of direct communication. Guided by the rationale that this can help consumers make better choices, governments have introduced regulations that penalize communicating deceptive or false claims about product quality attributes to buyers.[3]

[1] See, e.g., Baksi and Bose (2007) or Glaeser and Ujhelyi (2010).

[2] Dranove and Jin (2010) provide an excellent overview of theoretical and practical issues related to disclosure and its cost in different industries and the reasons why, despite regulatory efforts, firms or their certifiers may still lie.

[3] For instance, in the US, the FTC Policy Statement Regarding Advertising Substantiation states that "a firm's failure to possess and rely upon a reasonable basis for objective claims constitutes an unfair or deceptive act or practice in violation of Section 5 of the FTC Act" (see http://www.ftc.gov/bcp/policystmt/ad-decept.htm). In Canada, Section 74.01(1)(a) of the Competition Act prohibits any "representation to the public that is false or misleading in a material respect." In November 2016, the European Commission

In many markets, buyers do not (only) rely on direct communication by firms, but may also infer unobservable product quality from other observable market behavior of firms such as pricing.[4] Thus, it is important to examine the effect of regulation geared towards making direct communication more credible in a context where indirect communication through signaling is also possible. The key purpose of this paper is to understand the welfare effects of such regulation in markets where firms compete and where price signaling is an alternative mode of communication. We will show that while information regulation may not be useful in generating *more* information to buyers, it can affect market outcomes in unforeseen and unintended ways that deserve closer examination.

The basic framework we use for this purpose is a simple stylized model of a symmetric incomplete information Bertrand duopoly where products can be of either high or low quality and consumers have unit demand. A firm knows its own product quality, but the rival firm and the buyers do not observe it; this kind of information structure is relevant in markets where firms have "trade secrets" that are unknown to their competitors and where firms learn about their own quality attributes through internal product testing and development as well as through consumer complaints that are settled confidentially. In many of the examples cited above, such as the use of child labour and "fair" treatment of suppliers, the information is truly private to each firm.[5] High quality is more costly to produce, but generates more social surplus. Firms can use both pricing and direct communication to convey private information about their own product quality. Direct communication is costly and not necessarily credible as the information provided is not verifiable by buyers; firms may lie about their product quality. Firms simultaneously make their communication and pricing decisions; buyers use these to update their beliefs about true product qualities, and then make their purchase decisions.

We focus on how changes in the cost of lying affect the equilibrium structure and the market outcome. In the absence of a cost of lying (for instance, when there is no regulation that penalizes false claims), direct communication is not credible and firms engage only in price signaling; the latter also holds if the cost of direct communication is itself prohibitively high. Price signaling is associated with a welfare distortion as the low quality firm must

updated the 2006 Directive on Misleading and Comparative Advertising, making it unlawful to engage in false statements concerning product quality (see, http://ec.europa.eu/consumers/consumer_rights/unfair-trade/false-advertising/index_en.htm).

[4]There is a wealth of experimental and empirical evidence that sugggests that consumers base their subjective quality perceptions on prices and that firms use prices to signal "quality". See, among others, Monroe (1973), Rao and Monroe (1989), and Erdem et al. (2008).

[5]The model would be substantially different in case firms know each other's quality. In that case, a firm's actions are more difficult to interpret as consumers could make inferences on the basis of a firm's actions about the quality produced by the rival. Existing signaling models where firms have shared information get around this issue by making strong assumptions such as that it is common knowledge that one firm has low quality and the other has high quality (see, e.g., Caldieraro et al. (2011)).

earn sufficient rent to not mimic the high quality type's action. This implies that it must sell in the state of the world where the rival offers high quality. We term this distortion a *consumption distortion:* consumers buy the low quality product even though high quality is available and generates more surplus.[6] By penalizing false claims, claims of high quality become more credible. As a result, unless direct communication is too costly to be covered by high quality profit, even a moderate level of penalty for lying creates a strategic *individual* incentive for high quality firms to disclose their quality (truthfully) through direct communication in order to be more competitive. To gauge the welfare implication, the regulation induced cost of direct communication incurred by high quality firms must be weighed against the possible gain in social surplus due to more competitive pricing. We establish two broad set of results.

First, unless the cost of direct communication is prohibitively high, a moderate level of penalty for making false claims induces firms to use direct communication in order to be more competitive, but this increased competitive behavior does not eliminate the consumption distortion associated with pure price signaling; a moderate penalty for lying implies that after taking into account the penalty, low quality firms still have incentives to mimic the high type and so they need to earn sufficient rent which, as mentioned earlier, means that they sell to buyers even when the rival directly claims it sells high quality. The net effect is *competitive overdisclosure*, leaving society strictly worse off than with no regulation. In other words, moderate regulation is generally counterproductive.

Second, the desirability of a large penalty on false claims depends on the level of the direct communication cost. If this cost is relatively low, then welfare is maximized by imposing high penalties: high quality firms will then disclose their quality and will not be imitated by low quality firms even when the latter earn zero rent; the result is a very competitive market where low quality types engage in marginal cost pricing and are limit priced by high quality rivals eliminating all consumption distortion. However, as high quality firms themselves do not have significant market power, this equilibrium breaks down if the direct communication cost is itself somewhat large and cannot be covered by the high quality profit. In the latter case, for high quality firms to break even and engage in direct communication, low quality rivals must charge higher prices which is possible only if they sell in the state where the rival is high quality i.e., there is consumption distortion. Therefore, a penalty, no matter how large, will never eliminate the consumption distortion if the direct communication cost is beyond a minimum threshold level and in that case, it is optimal to have no regulation. Thus, a strong fine on false claims is desirable only if the direct communication cost is low; for intermediate levels of communication cost, regulating

[6]In markets where consumers have unit demand (such as the mortgage market or markets for electronic products) this is the only welfare distortion, but in markets where consumers have downward sloping demand there may also be a *quantity distortion* as price signaling is associated with relatively high prices.

false claims can only reduce welfare.

The general implication is that in designing information regulation, one needs to take into account the communication medium whose content is being regulated. Imposing a high penalty for deceptive communication when truthful communication is significantly costly may not be socially desirable. In addition, it may be difficult to impose high fines because of legal proportionality reasons: the fine should be proportional to the damage caused. In this case, even if direct communication is not costly, it may be better to have no fines than the maximum possible fines, if these latter are not able to reduce the consumption distortion.

Our analysis also suggests why we may expect industries to lobby unitedly against information regulation: an increased fine for lying often reduces profits earned by both low and high quality firms due to intensification of price competition (though it may increase the expected market share of high quality firms). Further, whenever high quality firms have an incentive to oppose regulation, it is in line with social welfare improvement.

Our model can be viewed as a multidimensional signaling model with competing senders where firms may use price and direct communication as signals. As the cost of the message (signal) is different, depending on whether it is truthful or not, an important difference with other signaling models is that the messages have literal meaning and cannot be simply interpreted as money burning.[7] To obtain a unique equilibrium outcome for a large part of the parameter space, we use the D1 refinement of beliefs, which selects the most competitive separating equilibrium. As mentioned above, by making false claims costly, regulation has a profound impact on the nature of equilibrium even though it does *not* lead to *more informed* consumer choices. Regulation affects the equilibrium outcome by modifying the beliefs that satisfy the D1 criterion. In particular, adverse out-of-equilibrium beliefs that deter high quality firms from cutting prices while claiming high quality become less reasonable when false claims are penalized more heavily. This makes the market more competitive.

The major qualitative results derived from our analysis continue to hold (i) with more than two quality types,[8] (ii) with weaker restrictions on beliefs of buyers, and (iii) in markets where quantity effects play a more important role. These extensions are discussed in Section 7 and details of the analysis are contained in the online appendix.

[7] In markets for experience goods with repeat purchase by buyers and where quality is persistent over time, a standard "money burning" argument (see, among many others, Nelson (1974), Milgrom and Roberts (1986)) suggests that using costly communication might be sufficient to convince buyers that the seller has a high quality product. In this paper, money burning plays no role as we focus on credence goods markets, and/or markets with high turnover of buyers and markets where quality varies over time due to exogenous factors.

[8] It is important for our analysis that firms' private information about quality is discrete and not continuous. In reality this information is likely to be very coarse for most products; the coarseness is created for instance by the technical limitations of product quality testing (which is often in terms of whether or not quality crosses certain thresholds). Other examples are risk categories for financial products, whether or not the production process meets an "organic", "green" or "non-GMO" test, whether or not child labor is used and whether users of pharmaceutical products report major, minor or no side-effects.

Our paper is related to several strands of literature. Technically, our model is based on Janssen and Roy (2010), where competing firms use prices to signal quality with no possibility of direct communication to buyers. Janssen and Roy (2015) analyze a duopoly model where firms may directly communicate their private information, but in that paper (as in much of the disclosure literature) direct communication is assumed to be fully credible and verifiable, i.e., firms cannot lie.[9] Further, direct communication is modeled as a strategic long-run decision that firms commit to prior to price competition to influence the information structure under which price competition takes place; indeed, in many of the pricing subgames, firms and consumers are fully informed about product quality through prior disclosure. Even if the penalty for lying is arbitrarily high, the results in this paper do not converge to those in Janssen and Roy (2015).[10]

In terms of content, our paper is related to a recently revitalized literature that aims to explain why false or deceptive advertising may occur in equilibrium. Thus, Rhodes and Wilson (2018) analyze a monopoly model where the production cost is independent of quality and consumers' out-of-equilibrium beliefs depend only on the disclosure statements, but not on price. They show that there exists an equilibrium where low and high quality firms choose the same actions and consumers may be deceived. The possibility of deceptive communication is also explored by Piccolo et al. (2015, 2017) in a duopoly model where it is known that one firm has high and the other has low quality, but buyers do not know which firm produces high quality. This literature is preceded by an earlier paper by Daughety and Reinganum (1997). They analyze a monopoly model where a firm may signal product safety through price and direct communication. They show that penalizing false claims can generate fully separating equilibria (no deception).[11] In contrast to this literature, the aim of our paper is *not* to explain when or why firms may engage in false advertising. To be clear, false claims can arise in our setting if we, like some of the above mentioned papers, assume that low and high quality are produced at the same cost or if we adopt weaker refinements of out-of-equilibrium beliefs; we discuss some of these possibilities in later sections. We focus on other, more subtle unintended consequences of information regulation by showing that *even if* firms do not make false claims in equilibrium, a policy of penalizing false claims may have a real impact on market outcomes through the beliefs that can be sustained.

The questions addressed in our paper can also be related to the literature on commu-

[9] See Daughety and Reinganum (2008b) for an analysis of disclosure and price signaling in a monopoly setting.

[10] For example, even if the direct communication cost is small, high quality firms may not want to disclose in the setting studied in Janssen and Roy (2015) as this intensifies price competition in the next stage. In our setting, high quality firms always disclose if the direct communication cost is small (and the penalty for lying is sufficiently large).

[11] They also present a duopoly version of that model that is quite different from the current paper in that quality is common between the firms and consumers incur a shopping cost.

nication with lying cost (see, e.g. Kartik, Ottaviani and Squintani 2007 and Kartik 2009) where it is shown that "inflated language" and incomplete separation of types are natural outcomes. An important difference with our setting is that we have multiple senders and we allow firms to have alternative means of signaling (through prices); further, both the sender's and the receiver's payoffs depend directly on one of the signals (price) that the competing senders (firms) choose. The two-dimensional nature of signaling and the fact that both the sellers' and buyers' payoffs depend directly on the price signal also distinguishes our framework from models of competition in Bayesian persuasion games (see, e.g., Gentzkow and Kamenica 2017, Li and Norman 2017 and Board and Lu 2018).

Finally, we briefly mention the large literature on signaling quality through price and/or advertising (see, among others., Milgrom and Roberts 1986, Bagwell and Riordan 1991, Daughety and Reinganum 2008a) or on quality disclosure (see, e.g., Grossman 1981 and Milgrom 1981). Unlike much of the disclosure literature, direct communication is not verifiable in our paper. In contrast to the advertising signaling literature, the messages cannot be simply interpreted as money burning.[12]

The rest of the paper is organized as follows. Section 2 presents the details of the model, the equilibrium concept and some basic properties of equilibria. Equilibria can be of three different types: (*i*) pure price signaling, (*ii*) a pure disclosure equilibrium where high quality firms always disclose, and (*iii*) mixed disclosure equilibrium where high quality firms randomize between disclosing and not disclosing their quality. Sections 3 - 5 fully characterize these equilibria. Section 6 combines all of these equilibria to understand the effect of penalizing false claims and the implications for optimal regulation. Section 7 discusses the extensions, while. Section 8 concludes and discusses some of the modeling assumptions. Proofs of key results are either outlined in the main text or contained in the Appendix; detailed proofs of other results as well as precise statements and proofs of additional results are contained in an online Appendix.

2 The Model and Preliminary Results

Consider a market with two firms, $i = 1, 2$, where each firm's product may be of either high (H) or low (L) quality. The true product quality is a firm's type which is purely private information: it is not known to the rival firm or to consumers. It is common knowledge that the *ex ante* probability that a firm's product is of high quality is $\alpha \in (0, 1)$ and independent across firms. The products of the firms are not differentiated in any dimension other than quality. Firms supply their output at constant unit cost c_s that depends on

[12] Our paper is also related to models where firms advertise content, such as Anderson and Renault (2006). In contrast to that paper, we have a competitive model (instead of a monopoly) with vertical (instead of horizontal) product differentiation where buyers learn about prices without having to incur a search cost.

quality $s \in \{H, L\}$, where $c_H > c_L \geq 0$.

There is a unit mass of identical consumers each having unit demand. A consumer's valuation of a product of quality s is given by V_s, where

$$V_H > V_L, V_s > c_s, s = L, H.$$

We focus on the more interesting case where the quality premium $V_H - V_L$ that buyers are willing to pay for the high quality product exceeds the cost difference:

$$\Delta V = V_H - V_L > \Delta c = c_H - c_L. \tag{1}$$

As high quality creates more social surplus than low quality, a *consumption distortion* is said to occur if consumers buy low quality even if high quality is provided by some firm in the market. It follows that there can only be a consumption distortion in the state of the world where one firm produces low and the other high quality, which arises with probability $2\alpha(1-\alpha)$.[13] For now, we will assume full market coverage (i.e., all buyers buy) and this is the case if

$$\Delta V \leq [(V_H - c_L)/2].^{14} \tag{2}$$

In Section 7, we show that there may be an additional *quantity distortion* if (2) does not hold and show that this does not radically change our welfare results.

Firms have the option to directly communicate their quality to all buyers by sending a message about their type. The (fixed) cost of direct communication is denoted by $D \geq 0$. Claims are *not* necessarily credible or verifiable and firms may lie about their quality. The parameter D captures the direct communication cost that is common to all firms. The differential cost of making false claims is denoted by $f \geq 0$. Like D, the parameter f depends on industry features (such as reputational concerns, or the cost of bribing third-party certifiers) and on regulation. If f is large, a firm would never lie and communication would effectively be credible and verifiable.

Formally, the game proceeds in three stages. First, nature independently draws the type (or quality) τ_i of each firm i from a distribution that assigns probabilities α and $1-\alpha$ to H and L respectively; the realization of τ_i is observed only by firm i. Next, both firms (having observed their own types), simultaneously choose their prices and messages to all buyers. Finally, consumers decide whether to buy and if so, from which firm. The payoff of each firm is its expected profit net of the direct communication cost and, where relevant, the

[13] Note that under complete information, there is no consumption distortion as a high quality producer would use its competitive advantage to reduce a low quality rival's market share to zero.

[14] The maximum prices high and low quality firms will set in any of the equilibria we study are obtained in the pure price signaling equilibrium where high (low) quality firms set prices equal to $c_L + 2\Delta V$, respectively $c_L + \Delta V$. Under this condition, these prices are smaller than or equal to V_H, respectively V_L.

fine. The payoff of each consumer is her expected net surplus.

The solution concept used is that of symmetric perfect Bayesian equilibrium (PBE) where the out-of-equilibrium beliefs satisfy a version of the D1 criterion (Cho and Kreps, 1987). In what follows, we simply refer to this as "equilibrium". The D1 refinement is useful to reduce the number of equilibria in the game, and for many parameter values we show that there is a unique D1 equilibrium. Roughly speaking, the D1 refinement selects the most competitive equilibrium in the set of fully revealing equilibria. However, the qualitative conclusions regarding policy implications remain true if a weaker refinement as the Intuitive Criterion (IC) is used. We elaborate on this in Section 7, where we discuss the welfare properties of pooling equilibria that are consistent with the IC. A comparative statics analysis under IC is more complicated due to the multiplicity of equilibria.

In principle, both low and high types could lie about their product quality. However, as D1 equilibria cannot be pooling (see below), in any equilibrium consumers will infer quality before buying. Therefore, in any D1 equilibrium a low quality firm will abstain from sending messages as this will save on communication cost and/or lying cost. Thus, we can restrict the messages m being chosen from the set $\{0, 1\}$ where 0 means no message and 1 means "I produce high quality".

The D1 criterion has been developed for signaling games with one sender and a one dimensional signaling space. Retaining tractability, a technical innovation this paper makes is to adapt the D1 criterion to environments with multiple senders and two signaling dimensions.[15] Consider firm i unilaterally deviating to a strategy (p, m) outside the support of its equilibrium strategy. As sellers' types are independently distributed, it is natural that buyers do not change their beliefs regarding the quality of the non-deviating player. Given the (possibly mixed) equilibrium strategies of the non-deviating firm, each profile of beliefs that buyers may possibly have about the type of firm i (following this deviation) and each profile of best responses of buyers (based on every such belief profile) defines a certain expected quantity sold by firm i at price p and message m. Let $B_i(p, m)$ be the set of all possible expected quantities sold by firm i at price p and message m that can be generated in this manner by considering all possible beliefs and best responses of buyers. Each $q_i \in B_i(p, m) \subset [0, 1]$ is a quantity that firm i can "expect" to sell at price p and message m for some profile of beliefs of buyers about firm i's type and for some configuration of optimal choices of buyers (that depend on realizations of prices charged by other firms) when other firms play according to their equilibrium strategy.

In the spirit of the D1 criterion, we compare the subsets of expected quantities in $B_i(p, m)$ for which it is gainful for different types of firm i to deviate to price p and message

[15] The notion of undefeated equilibrium, pioneered by Mailath, Okuno-Fujiwara and Postlewaite (1993) cannot be easily adapted to signaling games with multiple senders.

m. More precisely, consider any perfect Bayesian equilibrium where the equilibrium profit of firm i when it is of type τ is given by $\pi_{\tau}^{i*}, \tau = H, L$. Consider any $p \in [0, V_H]$ and $m \in \{0, 1\}$ outside the support of the equilibrium strategy of firm i and denote by $\pi_{\tau}^{i}(p, m; q_i)$ the profit firm i of type τ makes when he sells quantity q_i. For example, $\pi_{L}^{i}(p, 1; q_i) = (p - c_L)q_i - D - f$. If for $\tau, \tau' \in \{H, L\}$, $\tau' \neq \tau$,

$$\{q_i \in B_i(p, m) : \pi_{\tau}^{i}(p, m; q_i) \geq \pi_{\tau}^{i*}\} \subset \{q_i \in B_i(p, m) : \pi_{\tau}^{i}(p, m; q_i) > \pi_{\tau'}^{i*}\}$$

where "$\subset$ " stands for strict inclusion, then the D1 refinement suggests that the out-of-equilibrium beliefs of buyers (upon observing a unilateral deviation by firm i to price p) should assign zero probability to the event that firm i is of type τ and thus (as there are only two types) assign probability one to firm i being of type τ'. We use this extended D1 criterion in the rest of the paper.

2.1 General Properties of Equilibria

Having defined the equilibrium notion, we now show that any equilibrium is fully revealing. To see the main argument, suppose there exists a (partially) pooling equilibrium. In the discussion below, we will focus on a candidate equilibrium with direct communication, but a similar argument applies to other (partially) pooling equilibria. In such a candidate equilibrium, with at least some positive probability, low and high quality firms pool on claiming high quality and setting a price p and earn a profit of $q^e(p - c_L) - D - f$, and $q^e(p - c_H) - D$, respectively, where q^e is the expected quantity sold given these actions. As there is a strictly positive probability mass at p, it is easy to see that prices slightly above p must lie outside the support of the equilibrium strategy of both types. Thus, for $\varepsilon > 0$ sufficiently small, claiming high quality and setting a price $p + \varepsilon$ is an out-of-equilibrium action for both types. The critical quantity $\widehat{q}_{\tau}$ that makes a type τ firm indifferent between claiming high quality and setting price p and setting price $p + \varepsilon$ is given by

$$\widehat{q}_{\tau} = \frac{q(p - c_{\tau})}{p + \varepsilon - c_{\tau}}, \qquad i = L, H.$$

As $c_H > c_L$, $\widehat{q}_H < \widehat{q}_L$ so that the high quality type has an incentive to deviate for a wider range of quantity responses from buyers.[16] The D1 refinement then requires consumers to believe that it is the high quality firm that has deviated if they observe a price $p + \varepsilon$ with

[16]Note that if $\Delta c = 0$ a pooling equilibrium exists where consumers may be deceived by false claims of high quality. This clarifies the importance of the common cost assumption in Rhodes and Wilson (2015) and Piccolo et al. (2015, 2017) who argue that false advertising may occur in equilibrium. Also, pooling equilibria with false advertising exists if consumers always believe that both upward and downward price deviations come from low quality sellers, as in Piccolo et al. (2015, 2017), who do not consider equilibrium refinements.

disclosure.[17] If consumers would buy with some positive probability after observing price p and a claim of high quality and believing quality is average, these same consumers will certainly buy with a strictly higher probability at price $p+\varepsilon$ believing this price is set by a high quality firm. Thus, firms want to deviate to price $p+\varepsilon$ (while claiming high quality). An immediate implication of this argument is that in equilibrium the low quality type does not use direct communication to signal quality.

The next proposition summarizes:

Proposition 1 *There does not exist a pooling or partially pooling equilibrium. Low quality types do not use direct communication so no fine is incurred.*

As equilibria are fully revealing, consumers are not misled by the content of information disclosed by the firm even though firms have the option of lying. Fully revealing equilibria can be of three different types: (*i*) equilibria where high quality firms only rely on price signaling, (*ii*) equilibria where high quality firms always claim high quality directly and (*iii*) equilibria where high quality firms randomize between claiming and not claiming high quality. We call these equilibria, respectively, (*i*) pure price signalling, (*ii*) pure disclosure, and (*iii*) partial disclosure equilibria. The term disclosure is appropriate here as communication is always truthful when it occurs in equilibrium. In the next three sections, we study the properties of these equilibria.

3 Pure Price Signaling

To understand the role of direct communication and what regulation can achieve by making false statements more costly, it is important to understand how price signaling works and under what conditions firms abstain from direct communication. Janssen and Roy (2010) (hereafter, JR (2010)) analyze a version of our model where firms do not have an option to directly communicate quality. They characterize the unique (symmetric D1) pure price signaling equilibrium. If there is a pure price signaling equilibrium in our model, it must be identical to the one in JR (2010). In this equilibrium, low quality firms randomize their prices over an interval $[\underline{p}_L, \overline{p}_L]$ without mass point, with $\overline{p}_L = c_L + \Delta V$ and $\underline{p}_L = \alpha\overline{p}_L + (1-\alpha)c_L$ and distribution function F_L where

$$F_L(p) = 1 - \frac{\alpha}{1-\alpha}\left(\frac{\Delta V}{p-c_L} - 1\right) = \frac{1}{1-\alpha} - \frac{\alpha\Delta V}{(1-\alpha)(p-c_L)},$$

while the high quality firm sets a deterministic price $p_H^{ND} = \overline{p}_L + \Delta V$. By imitating the high quality price p_H^{ND} it makes a higher margin of ΔV, but only sells if the rival is of high

[17] Note that this argument nicely illustrates the idea that (even out-of-equilibrium) higher advertized prices signal higher quality.

quality and in that case only with probability $\frac{1}{2}$ and so it just does not have an incentive to imitate the high quality price. JR (2010) show that in their model there are multiple price signaling equilibria that do not satisfy the D1 refinement. These equilibria are identical to the D1 equilibrium up to the choice of $p_H^{ND} \geq c_L + 2\Delta V > c_H$. The D1 equilibrium selects the most competitive of these equilibria where the low quality firm is just indifferent between choosing a price $p \in [\underline{p}_L, \overline{p}_L]$ and imitating high quality and setting p_H^{ND}. Still, the equilibrium exhibits a fair amount of market power with $\overline{p}_L = c_L + \Delta V$, which is necessary to keep the low quality firm from imitating the high quality price.

Further, all consumers buy. If one firm charges p_H^{ND} and the other firm charges a price in $[\underline{p}_L, \overline{p}_L]$, buyers always buy from the latter; when both firms charge prices in $[\underline{p}_L, \overline{p}_L]$, buyers buy from the lower priced firm. Thus, in the price signaling equilibrium there is a consumption distortion in that when both low and high quality firms exist in the market, consumers buy low quality, even though the surplus generated by high quality consumption is larger. A high quality firm does not have an incentive to undercut as any firm charging price $p \in (\overline{p}_L, p_H^{ND})$, is believed to have low quality with probability one so that no consumer would buy at these prices as they would buy from the competitor instead.

As there is a consumption distortion, but no direct communication costs are incurred, the welfare loss, compared to the first-best, equals

$$WL = 2\alpha(1 - \alpha)(\Delta V - \Delta c). \tag{3}$$

Expected consumer surplus can be calculated as the difference between expected total surplus and firms' profit and is equal to

$$(V_L - c_L) - \alpha(2 - \alpha)\Delta V,$$

while a low quality firm makes a profit of $\pi_L^* = \alpha\Delta V$ as by charging $\overline{p}_L$ it only sells when the rival is of high quality (which happens with probability α) and if it sells it makes a margin of ΔV, whereas a high quality firm's profit equals $\pi_H^* = \alpha(\Delta V - \Delta c/2)$. It is clear that both total surplus, consumer surplus and firms' profits are independent of f and D whenever the pure price signaling equilibrium exists.

It remains to be seen for which parameter values D and f the pure price signaling equilibrium exists. The main reason why such an equilibrium may not exist is that the high quality firm may have an incentive to deviate by directly disclosing its quality if it is believed to be a high quality firm by consumers. Both the direct communication cost D and the fine f play an important role in this respect: D should not be too high for the deviation to be profitable, whereas f should not be too small for consumers to infer that a low quality firm would not have an incentive to engage in such a deviation.

To determine more precisely when a pure price signaling equilibrium exists, define two critical levels of the direct communication cost $\underline{D}$ and $\overline{D}$ as follows:

$$\underline{D} = \alpha(\Delta V - \frac{\Delta c}{2}) \text{ and } \overline{D} = \Delta V - (1 - \frac{\alpha}{2})\Delta c.$$

It is easy to check that $0 < \underline{D} < \overline{D}$. For each $D < \overline{D}$ define a critical level $f^*(D)$ as

$$\begin{aligned} f^*(D) &= \frac{D}{2\Delta V - \Delta c}\Delta c, \text{for } 0 \leq D \leq \underline{D} \\ &= \frac{D - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c}\Delta c, \text{ for } \underline{D} \leq D \leq \overline{D}. \end{aligned}$$

Observe that $f^*(D)$ is continuous and strictly increasing in D on $(0, \overline{D})$, $f^*(D) \to 0$ as $D \to 0$, and $f^*(\overline{D}) = (1 - \frac{\alpha}{2})\Delta c$.

The critical fine $f^*(D)$ defines the boundary of the region of pure price signaling and is determined by two conditions needed for a gainful deviation from the pure price signaling equilibrium not to be profitable. Depending on the parameter values, one of the two conditions is binding. A first condition is that if firms set a price $p \in (\overline{p}_L, p_H)$ and claim they are high quality, consumers do not believe them and instead believe it is the low quality firm that has deviated and falsely claims high quality. In this case, consumers do not buy, making the deviation unprofitable. A second condition says that even if there exist prices $p \in (\overline{p}_L, p_H)$ such that if they are associated with a claim of high quality, consumers (have to) believe that the deviation comes from a high quality firm, the high quality firm will nevertheless not find it profitable to set these prices. This is possible as it is at the higher prices in the interval that consumers believe the deviation comes from a high quality firm, but at these prices it may still be very unlikely that they will buy if a low quality firm is present as the low quality firm is randomizing and may well choose such a low price that the consumer still prefers to buy from the low quality firm. The first condition binds if D is smaller than $\underline{D}$ and the second condition binds if $D \in [\underline{D}, \overline{D})$.

Our main result in this section is that a pure price signaling equilibrium exists if, and only if, either (i) $D \geq \overline{D}$, or (ii) $D < \overline{D}$ and $f \leq f^*(D)$. If the direct communication cost D is too large (larger than $\overline{D}$), then it never pays to communicate quality directly even if the out-of-equilibrium beliefs are very favorable.

If the direct communication cost is smaller, then it may be gainful for a high quality firm to claim high quality (and undercut the high quality price), if this double signal is correctly interpreted (by consumers) as coming from a high quality firm. In terms of the D1 logic, this requires that a low quality firm should *not* have a stronger incentive to send such a message than a high quality firm. Intuitively, this depends on the fine f: if the fine for lying is large, then low quality firms will not have an incentive to send such a signal so that high quality firms will find it optimal to deviate; if f is small, then consumers will believe that low quality firms are trying to mislead them, and will therefore not buy from a firm sending this double signal, making it unprofitable for high quality firms to deviate.

Figure 1 depicts the region where a pure price signaling equilibrium exists. For ease of comparison with the characterization of equilibria in subsequent sections, the figure and the statement of the main result regarding pure price signaling equilibria is in terms of the function $D^*(f)$, which is the "inverse" of $f^*(D)$, where for each value of f $D^*(f)$ is the lowest direct communication cost at which a pure price signaling equilibrium exists:

$$D^*(f) = \begin{cases} (2\frac{\Delta V}{\Delta c} - 1)f, \text{ for } 0 \leq f < \frac{\alpha \Delta c}{2} \\ (\frac{\Delta V}{\Delta c} - 1)f + \frac{\alpha}{2}\Delta V, \text{for } \frac{\alpha \Delta c}{2} \leq f \leq (1 - \frac{\alpha}{2})\Delta c \\ \Delta V - (1 - \frac{\alpha}{2})\Delta c = \overline{D}, \text{for } f \geq (1 - \frac{\alpha}{2})\Delta c \end{cases}$$

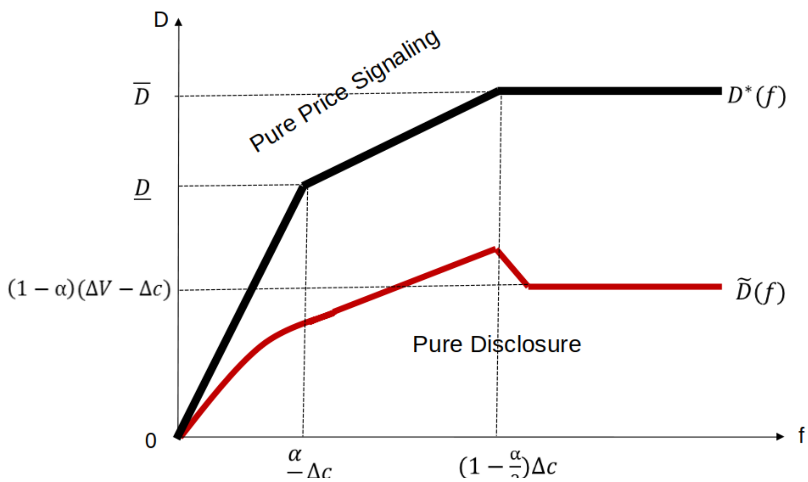

Figure 1: Regions of pure price signaling and pure disclosure equilibria

A symmetric pure price signaling equilibrium exists only to the left of and above $D^*(f)$.

Proposition 2 *A symmetric equilibrium with pure price signaling i.e., one where firms do not disclose directly, exists if, and only if, the direct communication cost* $D \geq D^*(f)$.

4 Pure Disclosure

In the previous section, we characterized the conditions under which the unreliability of direct communication about quality forces firms to only use prices to signal their private information. Price signaling is associated with a consumption distortion. One may expect that if the fine for making false claims is strengthened and direct communication itself is not too costly, firms will use direct communication (along with pricing) to credibly signal their private information and this may reduce the consumption distortion, possibly leading to welfare gains. In this section, we characterize the economic conditions under which high quality firms always disclose and examine the welfare implications.

4.1 Fully Distortionary Disclosure

We begin by showing that, contrary to what one might expect, even though moderate regulation can increase the credibility of direct communication to the extent that high quality firms always disclose, there is no correction of the consumption distortion so that the direct communication cost incurred by firms is a pure waste from a social point of view. High quality firms disclose with probability one in equilibrium, while low quality firms make positive profits, capturing the market even if high quality firms are present. Following the logic of the pure price signaling equilibrium, it is easy to see that low quality firms must randomize over an interval $[\underline{p}_L, \overline{p}_L]$ if they make positive profits. The pure disclosure equilibrium we characterize first is one where a high quality firm chooses a deterministic price p_H^D and $\overline{p}_L = p_H^D - \Delta V$.[18] The equilibrium is fully distortionary as direct communication does not reduce the consumption distortion observed in the pure price signaling case.

The main remaining issue is how to determine p_H^D. An important consideration is that low quality firms should not have an incentive to imitate high quality behavior. This implies a lower limit on p_H^D :

$$\frac{\alpha}{2}(p_H^D - c_L - 2\Delta V) + D + f \geq 0.$$

To determine p_H^D we have to consider the other major possibility for deviation, namely for any of the firms to set an out-of-equilibrium price $\widehat{p} \in (\overline{p}_L, p_H^D)$, whether or not accompanied by direct communication. It is clear that consumers will buy if they believe that a high quality firm has deviated, which makes such a deviation profitable. Using the D1 logic explained in the previous section, consumers will believe that a deviating firm that claims high quality and chooses $\widehat{p} \in (\overline{p}_L^D, p_H^D)$ is of low quality. This implies that a low quality firm should be indifferent between its equilibrium strategy and mimicking high quality behavior. This results in the requirement that

$$p_H^D = c_L + 2\Delta V - \frac{2(D + f)}{\alpha}. \tag{4}$$

Low quality firms make nonnegative profits if, and only if, $\overline{p}_L = p_H^D - \Delta V \geq c_L$ and this is

[18] The proof of Proposition 4 in the Appendix explains that there cannot be other equilibria where the high quality firm always discloses and the low quality firm makes positive profit.

guaranteed if, and only if,

$$D + f \leq \frac{\alpha}{2}\Delta V. \tag{5}$$

In the proof of the next proposition, we show that the requirement that consumers think that a deviating firm that does *not* claim high quality and chooses $\widehat{p} \in (\overline{p}_L^D, p_H^D)$ is of low quality is consistent with the D1 criterion if, and only if,

$$D \leq \left(\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1\right) f, \tag{6}$$

while both restrictions together imply that the high quality firms profits are nonnegative as well. Thus, we have

Proposition 3 *A fully distortionary pure disclosure equilibrium exists if, and only if, (5) and (6) hold, where high quality firms disclose and charge a deterministic price given by (4), while low quality firms do not claim high quality and randomize prices over the interval* $[\underline{p}_L, \overline{p}_L]$*, with* $\overline{p}_L = p_H - \Delta V > c_L$*. This equilibrium generates lower welfare than a pure price signaling outcome and is the unique equilibrium outcome in the region where (5) and (6) hold. Firms' profits are given by* $\pi_H^* = \frac{\alpha}{2}(p_H^D - c_H) - D$ *and* $\pi_L^* = \alpha(p_H^D - \Delta V - c_L)$.

The main difference between this equilibrium and the pure price signaling equilibrium is that the high quality firm sets a price that is $\frac{2(D+f)}{\alpha}$ lower than in the pure price signaling equilibrium, and that it uses direct communication (which is natural as the direct communication cost D is relatively low compared to the fine f). It immediately follows that *all* prices are lower than in the pure price signaling equilibrium. This causes consumer surplus to be larger and to be increasing in both D and f over the whole range where this equilibrium exists at the expense of firms' profits.

The situation with respect to welfare or total surplus is quite the opposite: as the consumption distortion is unaffected, while high quality firms engage in costly communication, the total welfare loss equals $[2\alpha(1-\alpha)(\Delta V - \Delta c) + 2\alpha D]$, which is larger than in the pure price signaling equilibrium.

The general implication is that if D is small, a fee f for lying does not necessarily eliminate the consumption distortion of the pure price signaling equilibrium and, in particular, an intermediate fine level is worse than no fines at all. In the next section, we will show that a similar statement holds true when D is larger.

4.2 Fully Non-Distortionary Disclosure

Next, we show that if the direct communication cost is small and regulation is strong enough, the equilibrium outcome involves direct communication, but unlike the equilibrium discussed above, the consumption distortion is fully eliminated and welfare improves relative to the pure price signaling case. In such a fully non-distortionary equilibrium, a low quality firm sells only where the competitor also sells low quality products; a Bertrand competition argument can then be used to argue that low quality firms earn zero profit and charge a deterministic price $p_L = c_L$. A high quality firm facing a low quality rival will only sell if his maximum price equals $c_L + \Delta V$. One can show that generically, high quality firms must randomize their prices to balance price competition with the high quality rival and the guaranteed market in the state where the rival is low quality.[19] It is clear that by charging the upper bound $\overline{p}_H^D = c_L + \Delta V$ the high quality firm will only sell where the competitor is of low quality, implying that it makes a profit of $(1-\alpha)(\Delta V - \Delta c) - D$. A first condition for this type of equilibrium to exist is that this profit is nonnegative, i.e.,

$$D \leq (1-\alpha)(\Delta V - \Delta c). \tag{7}$$

A second condition is that low quality firms should not have an incentive to imitate prices that are set in equilibrium by high quality firms. As the high quality firms are indifferent over a set of prices and as $c_H > c_L$ it follows that for the low quality firm the most profitable deviation is to deviate to the lowest price in the equilibrium support. In the proof of the next proposition we show that the low quality firm does not find it gainful to deviate to this price if, and only if,

$$D + f \geq (1-\alpha)\Delta V + \frac{\alpha}{2}\Delta c. \tag{8}$$

These arguments establish the key parts of the following proposition. The remaining elements of the proof are in the Appendix.

Proposition 4 *A fully non-distortionary (pure disclosure) equilibrium exists, if and only if, (7) and (8) hold. In this equilibrium, high quality firms disclose and (generically) randomize prices with* $\overline{p}_H^D = c_L + \Delta V$, *while low quality firms set* $p_L = c_L$, *and only sell if both firms produce low quality. This equilibrium generates higher welfare than a pure price signaling outcome, and is the unique equilibrium in the region of parameters defined by conditions (7) and (8). Firms' profits are given by* $\pi_H^* = (1-\alpha)(\Delta V - \Delta c) - D$ *and* $\pi_L^* = 0$.

Thus, a fine f on lying can fully eliminate the consumption distortion if it is sufficiently harsh and as long as D is sufficiently small. Further increasing the lying cost f has no effect

[19] High quality firms consistently setting a price $c_L + \Delta V$ can only be part of a non-distortionary equilibrium if consumers buy high quality in the state where both low and high quality firms are active. In that case, low quality firms do not have an incentive to imitate high quality firms if f is such that $D + f \geq \left(1 - \frac{\alpha}{2}\right)\Delta V$. To prevent high quality firms from undercutting, it should be the case, however, that consumers believe that prices lower than $c_L + \Delta V$ accompanied by a disclosure statement are set by a low quality firm. Using Proposition 2(b), this is consistent with the D1 criterion only if the low quality type is indifferent to imitating the high quality action which is satisfied only for a very specific value of f.

on the market outcome. It is clear that the market outcome is fairly competitive. Under full information, high quality firms set $p_H = c_L + \Delta V$ where there is one high and one low quality firm in the market. With asymmetric information, the high quality firm charges lower prices as with positive probability, it may be in competition with another high quality firm. Also, the high quality prices are strictly smaller and consumer surplus is strictly larger than in any of the other equilibria we have characterized so far. As there is no consumption distortion, the total welfare loss is constant at $2\alpha D$. As the parameter region is such that $D \leq (1-\alpha)(\Delta V - \Delta c)$, it is easy to check that this welfare loss is smaller than the welfare loss in the pure price signaling equilibrium.

4.3 Partially Distortionary Disclosure

Between the regions of a relatively low and high differential cost f of direct communication (and with the direct communication cost itself being relatively low), there is an intermediate region of parameter values where Propositions 4 and 5 do not hold. We now show that in this region the equilibrium outcome involves pure disclosure with partial distortion. In this equilibrium, high quality types disclose for sure and set a deterministic price $c_L + \Delta V$ while low quality firms set a price equal to c_L. Buyers buy from the firm charging price $c_L + \Delta V$ with probability $\beta \in (0,1)$ when the other firm charges a price equal to c_L. This equilibrium is partially distortionary as the consumption distortion arises with a positive probability $1-\beta$ if a low and high quality firm are present.

To guarantee that high quality firms do not have an incentive to undercut, it should be the case that consumers believe that if they observe a price p in the interval $(c_H, c_L + \Delta V)$ it is set by a low quality firm. Using the D1 logic explained in the previous section, this implies that the low quality firm should be indifferent between choosing $p_L = c_L$ and claiming high quality and setting $p_H = c_L + \Delta V$. Thus, as a low quality firm sells with probability $(1-\alpha)\beta + \frac{\alpha}{2}$ if it imitates the high quality price, we determine

$$\beta = \frac{D+f}{(1-\alpha)\Delta V} - \frac{\alpha}{2(1-\alpha)}. \tag{9}$$

For this equilibrium to exist, we should have that $0 < \beta < 1$, which translates into

$$\frac{\alpha}{2}\Delta V < D + f < \left(1 - \frac{\alpha}{2}\right)\Delta V, \tag{10}$$

and that $\pi_H^* \geq 0$, which implies that

$$D \leq \frac{\Delta V - \Delta c}{\Delta c} f. \tag{11}$$

These arguments establish the main parts of the following proposition.[20]

Proposition 5 *A partially distortionary pure disclosure equilibrium exists if, and only if, (10) and (11) hold. In this equilibrium* $p_L = c_L$*, and* $p_H = c_L + \Delta V$*, while consumers buy high quality with probability* β *given by (9). Firms' profits are given by* $\pi_H^* = \left[\frac{\alpha}{2}(1-\alpha)\beta\right](\Delta V - \Delta c) - D$ *and* $\pi_L^* = 0$*.*

It is easy to observe from (9) that β is increasing in D and f, i.e., the consumption distortion becomes smaller when one of these parameters increases. Further, the total (expected) direct communication cost that is incurred is constant at $2\alpha D$. Thus, the welfare loss decreases when f increases. As prices are such that consumers are indifferent between buying high and low quality, consumer surplus remains constant as f increases. High quality firms' profits are increasing in f , while low quality firms always make zero profit. Thus, in this parameter region, firms (weakly) favor more regulation and social welfare improves.

4.4 Overview of Disclosure Equilibria

Having characterized three types of equilibria, the next proposition states that for any $f > 0$ pure disclosure equilibria always exist if, and only if, D is smaller than a threshold value $\widetilde{D}(f)$ that is defined as

$$\widetilde{D}(f) = \begin{cases} \left(\frac{2\alpha\Delta V}{\alpha\Delta c + 2f} - 1\right) f \text{ , if } 0 \leq f \leq \frac{\alpha}{2}\Delta c \\ (\Delta V - \Delta c)\, f/\Delta c, \text{ if } \frac{\alpha}{2}\Delta c < f \leq (1-\frac{\alpha}{2})\Delta c \\ (1-\frac{\alpha}{2})\Delta V - f \text{ , if } (1-\frac{\alpha}{2})\Delta c < f \leq \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c \\ (1-\alpha)(\Delta V - \Delta c)\text{ , if } f > \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c. \end{cases}$$

It turns out that the equilibria characterized in Propositions 3 - 5 are the only pure disclosure equilibria. The function $\widetilde{D}(f)$ and the region of pure disclosure equilibria is also depicted in Figure 1.

Proposition 6 *A pure disclosure equilibrium (where high quality firms disclose with probability one) exists if, and only if,* $D \leq \widetilde{D}(f)$*.*

Note that for relatively large values of f, whether or not $\widetilde{D}(f)$ is large depends both on the difference in social surplus created by high and low quality $(\Delta V - \Delta c)$ and the probability quality is low, $1-\alpha$. Thus, it may well be that the direct communication cost D must be fairly small for this type of equilibrium to exist. Note also that $\widetilde{D}(f)$ is

[20] The remaining part of the proof is simple: a deviation to a price $p < p_H$ without disclosure is not gainful for either type if, and only if, the out-of-equilibrium beliefs assign sufficiently high probability to the firm being an L-type; as an L-type earns zero profit, such beliefs are always consistent with the D1 criterion.

non-monotone around $f = (1 - \frac{\alpha}{2})\Delta c$. In this region, and for higher values of f, $\widetilde{D}(f)$ is determined by the constraint that the high quality firm makes nonnegative profit. If f is relatively large, we have seen in Proposition 4 that the high quality firm randomizes with an upper bound on $c_L + \Delta V$ and only sells at this price if the competitor is a low quality type, which happens with probability $1 - \alpha$. If f is intermediate, however, the discussion above Proposition 5 shows that the high quality firm sets $c_L + \Delta V$ for sure and sells with probability $\beta(1 - \alpha) + \alpha/2$, where β is an endogenous parameter such that $\beta(1-\alpha)+\alpha/2 = (D+f)/\Delta V$. There are intermediate values of f such that $\beta(1-\alpha)+\alpha/2$ is larger than $1 - \alpha$ implying that for intermediate values of f high quality firms sell more than at higher values of f. Thus, they are willing to disclose also at higher values of the direct communication cost D. This makes $\widetilde{D}(f)$ non-monotone at intermediate f values.

5 Partial Disclosure

In the previous sections, we have discussed two kinds of equilibria: a pure price signaling equilibrium where firms never engage in direct communication about quality, and a class of equilibria where the high quality type discloses quality for sure. As illustrated in Figure 1, the regions of the parameter space where these equilibria exist are mutually exclusive; the pure price signaling equilibrium exists if, and only if, $D \geq D^*(f)$, whereas the pure disclosure equilibria exist if, and only if, $D \leq \widetilde{D}(f)$, with $\widetilde{D}(f) < D^*(f)$ for any f. As f approaches 0, both $\widetilde{D}(f)$ and $D^*(f)$ converge to 0. In this section, we characterize the equilibria for intermediate values of the direct communication cost where $\widetilde{D}(f) < D < D^*(f)$ and show that for these parameter values partial disclosure equilibria exist where high quality firms randomize between disclosing and not disclosing their private information. The main reason why high quality firms do not want to disclose for sure is that this would create too competitive a market where the firm(s) cannot recover their direct communication cost. Importantly, we show that in this region of the parameter space, welfare is always lower than under no regulation (pure price signaling) regardless of the fine.

We start the analysis when f is relatively small: $0 < f \leq \frac{\alpha\Delta c}{2}$. For these values of f, results in the previous two sections indicate that the pure price signaling equilibrium and the pure disclosure equilibrium look very similar to each other: the high quality firm sets a deterministic price and the low quality firm randomizes over prices, guaranteeing to sell to all consumers if the rival is of high quality. We now show that if $\widetilde{D}(f) < D < D^*(f)$ there exists an equilibrium that naturally transits between these two equilibria, namely a partial disclosure equilibrium where the high quality firm sets a deterministic price that is independent of whether or not it discloses, *i.e.*, $p_H^D = p_H^{ND} = p_H$, where if both firms set p_H and only one firm discloses, consumers buy from the disclosing firm with probability $\widetilde{\beta} > \frac{1}{2}$.

Thus, high quality firms randomize their communication decisions, relying on both signaling mechanisms we have encountered before. As the equilibrium is separating and consumers anyway infer that the firm is of high quality even if it does not communicate directly, they are indifferent between firms that charge p_H and hence may randomize their purchasing decisions. Because of the additional cost, it is clear that by communicating directly a high quality firm must sell more than a non-disclosing high quality firm, hence $\widetilde{\beta} > \frac{1}{2}$, to make the high quality type indifferent. The other features are the same as in the pure price signaling equilibrium and the pure disclosure equilibrium of Proposition 3.

The next proposition states the result more formally.

Proposition 7 *A fully distortionary partial disclosure equilibrium exists if, and only if,* $\widetilde{D}(f) < D < D^*(f)$ *and* $0 < f \leq \frac{\alpha \Delta c}{2}$. *In this equilibrium, high quality firms choose to directly disclose with probability* $0 < \gamma_H < 1$ *and always choose a price* $p_H = c_H + (D/f)\Delta c$, *while low quality firms randomize over the interval* $[\underline{p}_L, \overline{p}_L]$, *with* $\overline{p}_L = p_H - \Delta V$. *If both firms set* p_H *and one firm discloses, consumers buy from the disclosing firm with probability* $\widetilde{\beta} = \frac{1}{2} + \frac{f}{\alpha \Delta c}$. *This equilibrium generates lower welfare than a pure price signaling outcome and is the unique equilibrium outcome (under the conditions).*[21]

In terms of welfare properties, this equilibrium shares many features with the fully distortionary pure disclosure equilibrium in Proposition 3. The consumption distortion is at the same level as in the pure price signaling equilibrium, while high quality firms engage in wasteful disclosure with strictly positive probability (that approaches 1 as f increases to $\frac{\alpha \Delta c}{2}$). The total welfare loss $2\alpha\left[(1-\alpha)(\Delta V - \Delta c) + \gamma_H D\right]$ is higher than in the pure price signaling equilibrium. Also, the price charged by the high quality firm is decreasing in f, implying that *all* prices are decreasing in f (and lower than in the pure price signaling equilibrium). Consumer surplus is therefore increasing in f. Finally, profits of both types of firms are decreasing in f. Thus, in terms of profits and total surplus, this partial disclosure equilibrium is in between the pure price signaling and the pure disclosure equilibrium.

In the online Appendix, we show that a partial disclosure equilibrium also exists if $\widetilde{D}(f) < D < D^*(f)$ and $f > \frac{\alpha \Delta c}{2}$. Such an equilibrium also nicely transits between the pure price signaling equilibria and the pure disclosure equilibria in that, if it discloses, the high quality firm takes over the whole market and randomizes its prices with $\overline{p}_H^D = \underline{p}_L + \Delta V$ (eliminating all consumption distortion as in the pure disclosure equilibrium of Proposition 5), while if it does not disclose, it leaves the market to the low quality firm and sets $p_H^{ND} = \overline{p}_L + \Delta V = c_L + 2\Delta V$ thereby creating a consumption distortion and sufficient rent for the low quality firm as in the pure price signaling equilibrium. The randomization probability γ_H ranges between 1 and 0 when D increases from the upper bound $(1-\alpha)(\Delta V - \Delta c)$ of

[21] For a proof of this result, see R.1 of the proof of Lemma 2 in online appendix A.

the pure disclosure equilibria to D^* (the boundary of the pure price signaling equilibrium) and $\overline{p}_L$ moves accordingly from c_L to $c_L + \Delta V$.

Proposition 8 *If $f > \frac{\alpha \Delta c}{2}$ and $\widetilde{D}(f) < D < D^*(f)$, then there exists a partial disclosure equilibrium that is partially distortionary. When it does not disclose, a high quality firm charges $p_H^{ND} = \overline{p}_L + \Delta V$; when it does, it randomizes over low prices at which it appropriates the entire market from a low quality rival.*[22]

By leaving the market to low quality firms when it does not disclose, a high quality firm softens price competition, creating enough rent to cover the intermediate level of disclosure cost. As there is a positive probability that a high quality firm does not disclose and when it does so, cedes the market to the low quality competitor, the consumption distortion is never fully eliminated. Denoting by q_L^e, respectively $q_{L/D}^e$, the expected quantity sold by a low quality firm if the competitor is high quality, respectively a high quality disclosing firm, the expression for welfare loss in this partial disclosure equilibrium can be written as

$$WL = 2\alpha[\gamma_H D + (1-\alpha) q_L^e (\Delta V - \Delta c)].$$

This expression can be easily understood as follows. There is a probability of $(\alpha\gamma_H)^2$ that both firms are high type and incur direct communication costs D, while there is a probability of $2\alpha(1-\alpha\gamma_H)$ that one of the firms is a high quality firm and it will disclose with probability γ_H. Thus, the total expected direct communication cost equals $\left[2(\alpha\gamma_H)^2 + 2\alpha(1-\alpha\gamma_H)\gamma_H\right] D = 2\alpha\gamma_H D$. In a partial disclosure equilibrium the consumption distortion $(\Delta V - \Delta c)$ occurs when one firm is of low quality and the other is of high quality and the latter does not disclose, which occurs with a probability of $2\alpha(1-\alpha)q_L^e$.

Thus, as the welfare loss can be rewritten as

$$\begin{aligned} & 2\alpha[D\gamma_H + (1-\alpha)(\Delta V - \Delta c)\{(1-\gamma_H) + \gamma_H q_{L/D}^e\}] \\ = \quad & -2\alpha\gamma_H[(1-\alpha)(\Delta V - \Delta c)(1 - q_{L/D}^e) - D] + 2\alpha(1-\alpha)(\Delta V - \Delta c), \end{aligned}$$

it is strictly larger than that in the pure price signaling equilibrium if, and only if,

$$2\alpha\gamma_H[(1-\alpha)(\Delta V - \Delta c)(1 - q_{L/D}^e) - D] < 0.$$

As for any $\widetilde{D}(f) < D < D^*(f)$ we have $\gamma_H > 0$ and $D > (1-\alpha)(\Delta V - \Delta c)$, it follows that this inequality holds. In other words, for intermediate levels of the direct communication cost, no matter how large the fine f, the equilibrium outcome is welfare dominated by the pure price signaling equilibrium.

[22] For a proof of this result, see R.2 of the proof of Lemma 2 in online appendix A.

We conclude these three sections on equilibrium characterization by formally stating

Corollary 9 *An equilibrium always exists.*

Existence of a D1 equilibrium is non-trivial. The proof is made up out of the construction of equilibria for the various subsets of the parameter space, together with the observation that the union of the various subsets covers the whole parameter space. A symmetric D1 equilibrium is not always unique. The regions of partially distortionary partial disclosure equilibria and full disclosure equilibria overlap for D and f values such that $(1-\alpha)(\Delta V - \Delta c) < D < \widetilde{D}(f)$ and $(1-\frac{\alpha}{2})\Delta c < f \leq \frac{\alpha}{2}\Delta V + (1-\alpha)\Delta c$. There is no other overlap in the parameter regions for the different kinds of equilibria.

6 Regulatory Implications

Having characterized the equilibrium structure for the entire parameter space, we are now in a position to address the implications for optimal regulatory policy. In doing so, we also comment on the extent to which firms' incentives are aligned with total surplus or welfare. In particular, we start focusing on the comparative statics of changes in f on welfare and profits for different ranges of the direct communication cost $D < \overline{D}$.[23] As we have discussed in the introduction, regulation may also impact the direct communication cost D and therefore, at the end of the section, we will also comment on the comparative statics with respect to D.

In the absence of a differential cost of falsely claiming high quality ($f = 0$) only the pure price signaling equilibrium exists. As there is no direct communication cost, any disclosure by high quality firms can be imitated by low quality firms, and is therefore not believed by consumers. Initially, as regulation increases f, the equilibrium outcome remains unchanged. For somewhat larger values of f (in particular, when $f > f^*(D)$ as defined in Section 3) we transit to equilibria where direct communication occurs with strictly positive probability.

In a pure price signaling equilibrium, by ceding the entire market to the rival if he is of low quality, a high quality firm generates an expected welfare loss (conditional on his own type and relative to the first best) equal to $(1-\alpha)(\Delta V - \Delta c)$. If increasing f induces a high quality firm to disclose and correct some of this distortion, the maximum possible expected welfare gain (conditional on his own type) is given by $(1-\alpha)(\Delta V - \Delta c) - D$. So, if $D > (1-\alpha)(\Delta V - \Delta c)$ then direct communication never improves net welfare and the equilibrium outcome is at least weakly welfare dominated by the pure price signaling outcome that occurs if $f = 0$. Formally, we have:

[23] If $D > \overline{D}$ the pure price signaling equilibrium is the only equilibrium that exists, independent of the policy parameter f. As firms will rely on price signaling only, regulation does not have any effect.

Proposition 10 *If* $(1-\alpha)(\Delta V - \Delta c) < D < \overline{D}$*, any* $f > f^*(D)$ *generates strictly lower welfare than* $f = 0$ *and total surplus is maximized at* $f = 0$*.*

The proposition only compares welfare for high fines to welfare when fines are absent. For intermediate values of f the marginal effect of an increase in f on welfare depends on the specific values of D.

Figure 2 illustrates the effect on welfare loss of regulation that increases f for a given value of D that lies in the intermediate range. It shows that as we move beyond the region of pure price signaling, welfare loss is first (sharply) increasing in f before it decreases to a stable, relatively high loss for high values of f. As the cost of disclosure exceeds any surplus gain from correction of consumption distortion, higher disclosure tends to aggravate welfare loss; indeed, the increasing part of the graph reflects the increase in disclosure probability as f increases. However, increase in f also allows the disclosing high types to lower prices more aggressively to attract more market share from their low type rivals; this mitigates some of the welfare loss caused by more disclosure; the declining segment of the graph reflects this.

Next, consider the situation where the direct communication cost is small and in particular, $D < (\Delta V - \Delta c)\min\{\frac{\alpha}{2}, 1-\alpha\}$.[24] Using the argument outlined above, as $D < (1-\alpha)(\Delta V - \Delta c)$, one can see that if a fee f induces high quality firms to disclose for sure and sell to the entire market when facing a low quality rival (thereby fully correcting the consumption distortion associated with pure price signaling), then it strictly improves welfare over the pure price signaling equilibrium.

As regulation increases f until f equals $\frac{\alpha}{2}\Delta V - D$, there is no reduction in the consumption distortion. As high quality firms disclose for sure, the welfare generated is strictly lower than under no regulation and in fact, the welfare loss (compared to the first best) is at its highest at this level of f and equals $2\alpha[(1-\alpha)(\Delta V - \Delta c) + D]$. When f increases beyond $\frac{\alpha}{2}\Delta V - D$, welfare starts increasing; we first enter the region of the partially distortionary pure disclosure equilibrium described in Proposition 5 and the consumption distortion gradually disappears when f increases further as consumers shift to buying high quality with higher probability when both low and high quality firms exist in the market; the total welfare loss reaches $2\alpha D$ when $f = (1 - \frac{\alpha}{2})\Delta V - D$ at which point buyers buy only high quality as long as it is supplied in the market; at this high cost of making false claims, welfare is higher than if $f = 0$. When f increases beyond $(1-\frac{\alpha}{2})\Delta V - D$, equilibrium is fully non-distortionary and net surplus or welfare no longer changes. Thus, we have:

Proposition 11 *If* $D < (\Delta V - \Delta c)\min\{\frac{\alpha}{2}, 1-\alpha\}$*, it is optimal to impose a fine* $f \geq$

[24] If $\frac{\alpha}{2} < 1-\alpha$, the comparative statics for $D \in [\frac{\alpha}{2}(\Delta V - \Delta c), (1-\alpha)(\Delta V - \Delta c)]$ is a somewhat complicated combination of the comparative statics of the two regions of direct communication cost analyzed here (as there may be several transitions between the pure disclosure and partial disclosure equilibria).

$(1-\frac{\alpha}{2})\Delta V - D$. *An intermediate level of the fine, i.e.,* $f^*(D) < f < \frac{\alpha}{2}\Delta V - D$, *is strictly worse than no fine.*

Figure 3 illustrates the welfare effects presented in the proposition. For a fixed low value of direct communication cost D, it shows the non-monotonic effect of regulation that increases f on welfare loss; welfare is at a minimum for intermediate values of f and maximized once f is large enough.

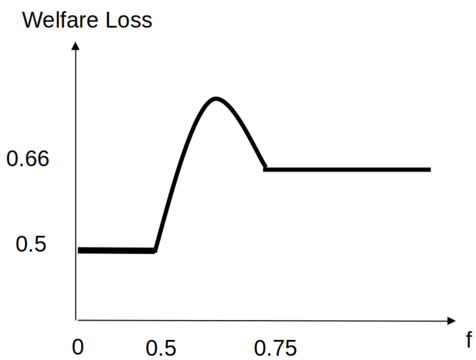

Figure 2: Effect of Regulation on Welfare Loss for Intermediate Cost of Direct Communication: $\Delta V = 2, \Delta c = 1$, $\alpha = 0.5, D = 1$

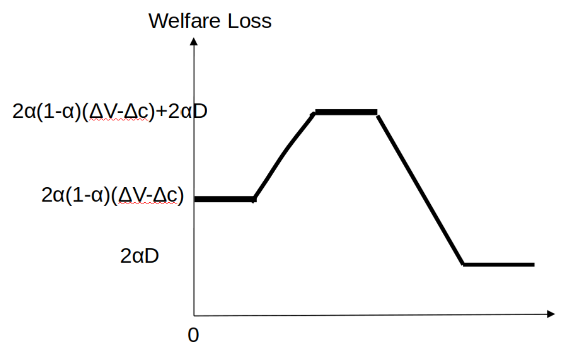

Figure 3: General effect of Regulation on Welfare Loss for Low Cost of Direct Communication

It is interesting to ask whether firms have an incentive to lobby in favor of or against making false claims costly. Indeed, at first glance, one may think that a penalty on lying about product quality should hurt low quality firms and possibly benefit high quality firms. What we find, however, is that for a segment of the parameter space, penalizing false claims affects the profits of high and low quality firms in the same direction and that the interests of both types of firms are broadly aligned with welfare.

The region where one can most clearly see this is the fully distortionary partial disclosure equilibrium described in Proposition 7 that arises if f is increased beyond $f^*(D)$. Proposition 7 claims that in the relevant region high quality firms price at $p_H = c_H + (D/f)\Delta c$. It is clear that this price is decreasing in f. As low quality firms price over an interval that moves one-to-one with p_H, it is clear that low quality prices and profits are decreasing in f as well. High quality profits also decrease in f as can be easily seen by substituting the expressions for p_H and $\widetilde{\beta}$ in $\pi^*_H = \alpha\widetilde{\beta}(p_H - c_H) - D$. Both types of firms also make less profits than in a pure price signaling equilibrium. So, if there is a proposal to create or marginally increase the fine on false claims, both high and low quality firms lobby against it. The fact that even high quality firms may be opposed to penalizing low quality firms for lying is based on the desire to protect the market power created by the use of price signaling and the pro-competitive effects of making direct communication more credible. Further, as noted above, the total surplus generated with such regulation is always smaller than under no regulation so that an industry lobby would be aligned with social welfare.

A similar argument holds true if $D > \frac{\alpha}{2}(\Delta V - \Delta c)$ and the fine f is further increased so that we transit to the equilibrium outlined in the online Appendix, or if the direct communication cost is small and we consider an intermediate level of the fine f. In the latter case, if the fine f is increased we transit from the fully distortionary partial disclosure equilibrium of Proposition 7 to the fully distortionary pure disclosure equilibrium of Proposition 3 where the profits of both types decline with f until the low quality firm earns zero profit at $f = \frac{\alpha}{2}\Delta V - D$. Both types of firms would lobby against this "intermediate" level of regulation which, as noted above, also lowers welfare compared to the pure signaling equilibrium.

7 Extensions

In this section, we consider three extensions. First, we consider a version of the model where there are more than two quality grades. Next, we analyze the case where (2) does not hold and quantity effects due to price changes arise. Third, we consider the consequences of weakening the D1 refinement criterion; in particular, we do a welfare analysis of pooling equilibria that satisfy the Intuitive Criterion. More details on the results of this Section are given in online Appendices B and C.

7.1 More than two quality types

Consider an extension of the model to multiple qualities. To focus ideas, assume that there are three quality types, denoted by L (low), M (medium) and H (high) with prior probability $\alpha_\tau \in (0,1), \alpha_H + \alpha_M + \alpha_L = 1$. Buyers have unit demand with valuation V_τ for quality τ. A firm with product quality τ can supply the good at constant unit cost $c_\tau \geq 0, \tau = L, M, H$ where

$$V_H - c_H > V_M - c_M > V_L - c_L > 0$$

and

$$c_H > c_M > c_L \geq 0.$$

Each firm can directly communicate a message about its product quality by incurring a cost $D > 0$; a message can be any subset of $\{L, M, H\}$; if the true product quality of the firm is not an element of the message sent, the firm incurs an additional fine $f \geq 0$.

The main qualitative results derived for the case of two types continue to hold for this three types case. In particular, we can show the following:

1. If f is small (very weak regulation) and/or D is large (costly disclosure), pure price signaling equilibria satisfying the D1 criterion exist. No pure price signaling equilibrium exists when f is large and D is not prohibitively large.

2. A fully non-distortionary (pure disclosure) equilibrium exists if f is large (strong regulation) and D is small enough. This equilibrium generates higher welfare than a pure price signaling outcome.

3. If regulation is not strong enough (f is not too high), then for small D there exists a pure disclosure equilibrium with full distortion. The welfare generated in such an equilibrium is lower than a full distortion pure price signaling outcome.

4. For large enough f a mixed disclosure equilibrium exists for intermediate values of D. Such an equilibrium generates lower surplus than the worst possible pure price signaling equilibrium under certain additional parametric restrictions.

Together, these results show that if D is small, welfare is non-monotonic in f, the fine for lying, but total surplus is maximized when f is large. When D is intermediate, high levels of the fine for lying can lead to mixed disclosure equilibria that are worse than the pure price signaling equilibria. The proofs of these results indicate that similar results should continue to hold for any finite number of quality grades. However, the characterization with more types can be considerably more cumbersome.[25] We now discuss these results in more detail.

With three types, pure price signaling equilibria come in three different variants, all extending the two type price signaling equilibrium in a natural way. In all these equilibria low quality firms randomize and high quality firms choose a pure price strategy, as in the two-type model. The three variants differ in whether or not the medium quality type randomizes and whether or not all consumers buy medium quality in the state where only medium and high quality products are available in the market.

That pure price signaling equilibria continue to exist when f is small or D is large, is not surprising. If D is large, direct communication is too costly and when f is small, a message can (and will) be imitated without large additional cost. On the other hand, if f is large enough and D is small, higher quality types want to directly communicate and combine this with lower prices to gain market share from potential lower quality rivals.

The second result extends Proposition 4 showing that if D is small and f is large, the consumption distortion is fully eliminated and welfare improves relative to pure price

[25] One may wonder whether a model with a continuum of types may be easier to work with. In online Appendix B, we show, however, that equilibrium may not exist with a continuum of possible qualities. As we have argued in the Introduction, for most real-world applications, it is more reasonable to think of private information about measurable quality as being discrete.

signaling. The third result extends Proposition 3 by showing that for intermediate values of f there exist equilibria that have the same consumption distortion as the full distortion pure price signaling equilibria but where firms incur communication costs. Together these two results demonstrate that for small values of the communication cost D welfare is non-monotone in f. Our fourth result extends Proposition 8 and 10 by showing that for larger values of D where mixed disclosure equilibria exist, high values of the fine f can be worse than no fine at all.

There are a few new insights from our analysis of the model with three (or more) qualities. First, there may be pure price signaling outcomes where some buyers buy the better of the two qualities available in the market; the high quality firm may sell (though to no more than half the buyers) when facing a medium quality rival thus mitigating some of the potential consumption distortions even if there is no information regulation. However, significant distortions remain. Second, D1 does not guarantee uniqueness of equilibrium as different varieties of pure price signaling equilibria may coexist for the same parameter values. Third, it is not the case that mixed disclosure equilibria always lead to lower welfare than pure price signaling equilibria. In particular, if D is relatively small and (for large f) close to the region where higher types always go for direct communication, welfare may be higher. Fourth, although firms may, in principle, understate their true quality and send vague, but truthful messages, the D1 refinement eliminates such equilibria. The main intuition is that if in a D1 equilibrium a higher quality type understates his true quality in a vague message, he may deviate to undercut the candidate equilibrium price and communicate his true quality precisely; due to lying cost, such a deviation would be more costly for lower types and so consumers would infer this deviation as coming from the higher type, making the deviation gainful.

7.2 Quantity Distortions

In this extension, we show how the general welfare results for the case where (2) holds and where there is no quantity effect carry over to the case where there is a quantity effect. If (2) does not hold, the cost of direct communication and the differential cost of making false claims may not only divert market share to the high quality type (when facing a low quality rival), but also reduce the quantity distortion. Below, we will show that penalizing false claims is welfare improving for a larger section of the parameter space, but it remains true that it is not welfare improving for other parameter values.

Lemma 2 in Janssen and Roy (2010) shows that in this case in a symmetric pure price signaling equilibrium $p_H = V_H$ and $\overline{p}_L = V_L$ and that in the state where both firms are high quality, only a fraction $\eta^S = 2\left(\frac{V_L - c_L}{V_H - c_L}\right) < 1$ of buyers buy in the unique symmetric D1 price signaling equilibrium. The smaller η^S the larger the inefficiency due to a *quantity* *distortion.* This quantity distortion is the closest one can get in a model with unit demand to the quantity effect due to price competition in a model with downward sloping demand. The closer α is to 1, the quantitatively more important the quantity distortion is as in that case it is most likely that both firms produce high quality.

It is easy to check that for any $D > 0$ the price signaling equilibrium with η^S is the unique equilibrium of the model with disclosure if f is small enough or, equivalently, for any $f \geq 0$ if D is large enough. The welfare loss in the pure price signaling equilibrium is given by:

$$WL^S = 2\alpha(1-\alpha)(\Delta V - \Delta c) + \alpha^2(1-\eta^S)(V_H - c_H), \tag{12}$$

where in addition to the consumption distortion we have encountered before (and represented by the first term), we have the quantity distortion that a fraction of $1-\eta^S$ consumers do not buy in the state where both firms produce high quality (which is reflected by the second term).

We proceed then with a few observations. First, it continues to be true that if $D \leq (1-\alpha)(\Delta V - \Delta c)$ then a sufficiently high value of f is optimal. In Section 4, we concluded that if (2) holds, $\bar{p}_H = c_L + \Delta V < V_H$ for sufficiently high values of f. The conditions for this equilibrium to exist are unaffected. Thus, the welfare conclusion continues to hold if (2) does not hold.

Second, Section 5 shows that under full consumer participation if the direct communication cost is in an intermediate range then a large fine on false claims generates strictly lower welfare than in the absence of such a fine. We now explore how this result is modified if (2) does not hold. Consider the pure price signaling equilibrium if (2) does not hold, described above. Let $\widehat{D}$ be the maximum gain to a high quality firm that deviates from a pure price signaling equilibrium by disclosing and reducing the price to $\underline{p}_L + \Delta V$ at which it sells to all buyers with probability one. Then, as $\underline{p}_L = \alpha V_L + (1-\alpha)c_L$

$$\begin{aligned}\widehat{D} &= (\underline{p}_L + \Delta V - c_H) - \frac{\alpha\eta^S}{2}(V_H - c_H) \\ &= (1-\alpha)(\Delta V - \Delta c) + \alpha[1 - \frac{\eta^S}{2}](V_H - c_H).\end{aligned}$$ [26]

For $D \geq \widehat{D}$, an equilibrium outcome is necessarily one with pure price signaling. In the online Appendix we show that:

(i) as in Section 5, a partial disclosure equilibrium exists if f is large and the direct communication cost $D \in ((1-\alpha)(\Delta V - \Delta c), \widehat{D})$;

(ii) unlike Section 5, these partial disclosure equilibrium outcomes are better than the pure price signaling outcome if the direct communication cost D is at the lower end of this range, *i.e.*, close to $(1-\alpha)(\Delta V - \Delta c)$;

(iii) as in Section 5, the partial disclosure equilibrium outcomes are worse than the pure price signaling outcome if the direct communication cost D is at the upper end of this range, *i.e.*, close to $\widehat{D}$.

Given the discussions in earlier sections, these results should be quite intuitive as a high fee f creates a more competitive market, and this may (partially) eliminate the quantity distortion associated with pure price signaling. In particular, together with the fact that if $D \leq (1-\alpha)(\Delta V - \Delta c)$ a sufficiently high value of f remains optimal, part (ii) says that the range where a high fee of falsely claiming high quality is larger than before due to the elimination of the quantity distortion. Thus, the region where the pure price signaling equilibrium generates higher welfare is smaller than in previous sections. However, part (iii) says that there remains a range of parameter values where it is optimal to have no fine.

These main effects can be explained as follows. The welfare loss in this partial (mixed) disclosure equilibrium with a quantity distortion is equal to:

$$WL^M = 2\alpha\gamma_H D + 2\alpha(1-\gamma_H)(1-\alpha)(\Delta V - \Delta c) + \alpha^2(1-\gamma_H)^2(1-\eta^S)(V_H - c_H). \quad (13)$$

This expression is similar to the expression for the welfare loss in a partial disclosure equilibrium, which we have seen in Section 5. There are two differences. First, if f is large, the expected quantity sold by a low quality firm if the competitor is a high quality disclosing firm $q^e_{L/D} = 0$. Second, as expected, there is an additional term reflecting the quantity distortion.

Thus, using (12), $WL^M - WL^S \leq 0$ if, and only if,

$$D \leq (1-\alpha)(\Delta V - \Delta c) + \alpha(1 - \frac{\gamma_H}{2})(1-\eta^S)(V_H - c_H). \quad (14)$$

As γ_H decreases from 1 to 0 if D increases from the lower bound $(1-\alpha)(\Delta V - \Delta c)$ to $\widehat{D}$, it is easy to see that the partial disclosure equilibrium generates more social surplus (a smaller welfare loss) if D is larger than, but still close to, $(1-\alpha)(\Delta V - \Delta c)$. On the other hand, as D gets close to $\widehat{D}$ and γ_H gets close to 0 it is easy to see that the right hand side of (14) is smaller than $\widehat{D}$ so that for direct communication cost D close to $\widehat{D}$ a large fine f generates lower social surplus than no fine.

7.3 (Deceptive) Pooling Equilibria

One may wonder to what extent the welfare results depend on the use of the D1 refinement. In particular, one may be interested to what extent high fines for deceptive claims to high quality may be optimal as they may eliminate an undesirable deceptive pooling equilibrium. In this section, we show that the more important qualitative welfare conclusions outlined before continue to hold if we also consider pooling equilibria with and without deception.

In particular, we argue that if D is small, a high value of f continues to be welfare optimal, whereas if D is relatively large, a small fine f is better than a large fine. To investigate this issue we consider, in this subsection, a wider class of equilibria that satisfy a weaker restriction on out-of-equilibrium beliefs, namely the Intuitive Criterion (IC). Under the IC, pooling equilibria with and without deception and with different price levels as well as less competitive separating equilibria exist. Given the large set of equilibria, it is difficult to perform a comparative statics analysis for all values of f and D, but comparing welfare for extreme values of f remains possible.

We first consider when pooling equilibria satisfying IC exist. First, it is clear that deceptive pooling equilibria with claims to high quality do not exist if D and f are relatively large. In particular, as in a deceptive pooling equilibrium with price $\widetilde{p}$, high and low quality types make a profit of $(\widetilde{p}-c_H)/2 - D$, respectively $\widetilde{p}/2 - D - f$, whereas for consumers to buy, $\widetilde{p}$ cannot be larger than $\alpha V_H + (1-\alpha)V_L$, so we need that $D < (\alpha V_H + (1-\alpha)V_L - c_H)/2$ and $D + f < (\alpha V_H + (1-\alpha)V_L)/2$. Second, we claim that a non-deceptive pooling equilibrium with price $\widetilde{p}$ exists, if and only if, $D \leq (1-\alpha)\Delta V$. To see this, note that the equilibrium payoffs are $\widetilde{p}/2$ and $(\widetilde{p} - c_H)/2$ for low, respectively, high quality and consider a deviation where the deviating firm claims high quality and sets a price $p' \neq \widetilde{p}$ such that $V_H - p' > \alpha V_H + (1-\alpha)V_L - \widetilde{p}$. The IC says that consumers should believe this deviation comes from a high quality firm if $p' - D - f < \widetilde{p}/2$ and $p' - c_H - D > (\widetilde{p} - c_H)/2$ as these conditions guarantee that the deviation cannot be profitable for a low quality type, but it can be profitable for a high quality type. Thus, a high quality firm would want to deviate if there exists a deviation price p' satisfying

$$c_H + D + (\widetilde{p} - c_H)/2 < p' < \min\{D + f + \widetilde{p}/2, (1-\alpha)\Delta V + \widetilde{p}\}.$$

For high enough values of f, one can always find a deviation price p' that satisfies this condition if $\widetilde{p} > c_H + 2D - 2(1-\alpha)\Delta V$. It thus follows that a non-deceptive pooling equilibrium does not exist for high enough values of f if $D \leq (1-\alpha)\Delta V$ as in that case any pooling equilibrium price must be such that the high quality firm has an incentive to deviate. On the other hand, if $D > (1-\alpha)\Delta V$ a non-deceptive pooling equilibrium with $\widetilde{p} = c_H$ exists for all values of f.

We next consider the welfare loss compared to the first best. It is clear that the welfare losses for the pooling equilibria without, respectively, with deception are equal to $\alpha(1-\alpha)(\Delta V - \Delta c)$ and $\alpha(1-\alpha)(\Delta V - \Delta c) - 2D - 2(1-\alpha)f$. In a pooling equilibrium, the consumption distortion only arises in half of the cases when there is a high and low quality firm in the market as consumers may buy high quality by chance.

With these intermediate results, the two welfare claims made in the beginning of this section easily follow. First, if D is small, a high value of f is welfare optimal. In particular, if $D \leq (1-\alpha)\Delta V$ the pure disclosure equilibrium we have characterized in Section 4 is the only equilibrium configuration that satisfies the Intuitive Criterion. This is because no pooling equilibria exist, whereas high profit revealing equilibria cannot be sustained either as they rely on consumers believing deviations with claims to high quality come from low quality firms. The welfare loss in this equilibrium equals $2\alpha D$,which is strictly smaller than the welfare loss in any of the pooling equilibria if

$$2D < (1-\alpha)(\Delta V - \Delta c).$$

Thus, for small enough values of D, it remains optimal to impose a large fine f even if we use a weaker equilibrium notion that allows for (deceptive) pooling equilibria.

Second, for large values of D, a small fine is better than a large fine. First, we have seen above that a non-deceptive pooling equilibrium exists for any value of f if $D > (1-\alpha)\Delta V$. Second, a deceptive pooling equilibrium does not exist for any f if $D > (\alpha V_H + (1-\alpha)V_L - c_H)/2$. In addition, other types of deceptive equilibria (where low quality types (partially) reveal their type through the prices they set) do not exist either. Thus, if

$$\max\{(1-\alpha)\Delta V, \frac{\alpha V_H + (1-\alpha)V_L - c_H}{2}\} < D < \overline{D} = \Delta V - (1-\frac{\alpha}{2})\Delta c$$

a small f is the (weakly) optimal choice: a small f eliminates the partial disclosure equilibria (which as we have seen in Section 5 is worse than the pure price signaling equilibrium that exists if $f = 0$), while the choice of f does not affect whether or not pooling equilibria exist.

8 Conclusion

Regulations that affect the costs associated with communication of product attributes by firms can have important and unintended consequences on market outcomes. we present a tractable framework that allows us to capture strategic competition and multidimensional signaling to analyze the welfare effects of regulation. In markets where firms can also signal their private information through price, direct communication is only one of the channels through which product information may be conveyed. While penalizing false claims can improve the credibility and increase the use of direct communication, it will also alter the conduct of firms in other dimensions such as pricing. In particular, penalizing false claims makes the market more competitive and reduces consumption and quantity distortions associated with signaling. However, improving the credibility of direct communication can lead to excessive disclosure in a competitive setting. Whether or not penalizing false claims is socially desirable critically depends on the level of direct communication cost.If the direct communication cost is small, imposing a sufficiently high fine for falsely claiming high quality is welfare optimal. The welfare effects are, however, non-monotonic and an intermediate fine is worse than no regulation. If the direct communication cost is larger, no regulation regarding false claims is socially optimal as high quality firms will not always claim they sell high quality as they would not be able to recover their direct communication cost in the competitive market that would result if they were to directly communicate their quality.

Competition plays an important role in our main results. In the monopoly version of our model, the incentives of the high quality firm to disclose are aligned with that of a social planner and it is always socially optimal to impose a high penalty on false claims. Our analysis indicates that deriving policy conclusions by ignoring the competitive interaction of firms and the possibility of excessive communication can be misleading.

APPENDIX

Proof of Proposition 2.

We focus on a candidate pure price signaling equilibrium as described in the text preceding the proposition. To establish the existence of such an equilibrium, we fix out-of-equilibrium beliefs of buyers to be such that any firm that does *not* directly communicate and deviates to a price in $(\overline{p}_L, p_H)$ is perceived as being a low quality type for sure. As this has been shown to be a D1 perfect Bayesian equilibrium in the special case where firms are not allowed to disclose (Janssen and Roy, 2010), to show that this is an equilibrium in our model the main step is to check that neither type has any incentive to deviate by *directly communicating* and charging a price $\widehat{p} \in (\overline{p}_L, p_H^{ND})$. Let $q_H(\widehat{p})$,$q_L(\widehat{p})$ be the expected quantity that an H and an L type firm must sell respectively in order to be indifferent between this deviation and not deviating from their equilibrium strategies:

$$q_H(\widehat{p}) = \frac{\pi_H^* + D}{(\widehat{p} - c_H)}, q_L(\widehat{p}) = \frac{\pi_L^* + D + f}{(\widehat{p} - c_L)}$$

Note that $q_H(\widehat{p}) \geq q_L(\widehat{p})$ if, and only if,

$$\frac{\widehat{p} - c_L}{\widehat{p} - c_H} \geq \frac{\pi_L^* + D + f}{\pi_H^* + D} \tag{15}$$

and as the left hand side of (15) is continuous and strictly decreasing in $\widehat{p}$, (15) holds for all $\widehat{p} \in (\overline{p}_L, p_H^{ND})$ if, and only if, (15) holds for $\widehat{p} = p_H^{ND}$ which (using the fact that $\pi_H^* = \frac{\alpha}{2}(p_H^{ND} - c_H)$ and $\pi_L^* = \alpha\Delta V = \frac{\alpha}{2}(p_H^{ND} - c_L)$) reduces to

$$f \leq \frac{D}{2\frac{\Delta V}{\Delta c} - 1}. \tag{16}$$

Under (16), $q_H(\widehat{p}) \geq q_L(\widehat{p})$ so that the D1 refinement is consistent with beliefs associating disclosure and price $\widehat{p} \in (\overline{p}_L, p_H^{ND})$ as coming from an $L-$type firm with probability one so that the candidate pure price signaling equilibrium is a D1 equilibrium in our model. Next, suppose that (16) does not hold. Then, one can check that there exists a unique $p_0 \in (c_H, p_H)$ such that (15) holds with equality at $\widehat{p} = p_0$ and, in particular,

$$p_0 = \frac{(\alpha \Delta V + A + f)\, \Delta c}{f + \frac{\alpha}{2} \Delta c} + c_L \tag{17}$$

and D1 refinement requires that a firm deviating to disclosure and price $\widehat{p} \in (p_0, p_H^{ND})$ should be regarded as an H-type with probability one. It is easy to see that if $p_0 \geq \underline{p}_L + \Delta V$, a deviating firm will not set a price below $\underline{p}_L + \Delta V$ because the firm sells to all buyers with probability one at price $\underline{p}_L + \Delta V$. Without loss of generality, we confine our attention to deviations to a price $\widehat{p}$ satisfying

$$\widehat{p} \geq r = \max\{p_0, \underline{p}_L + \Delta V\}. \tag{18}$$

Deviation to any price $\widehat{p}$ satisfying (18) is strictly gainful for a type H firm if, and only if,

$$\begin{aligned} \pi_H^* &< (\widehat{p} - c_H)[\alpha + (1-\alpha)(1 - F(\widehat{p} - \Delta V))] - D \\ &= \left[\frac{\widehat{p} - c_H}{\widehat{p} - \Delta V - c_L}\right] \alpha \Delta V - D \end{aligned} \tag{19}$$

and (as the right hand side of (19) is strictly decreasing in $\widehat{p}$) this holds if, and only if,

$$\pi_H^* < \left[\frac{r - c_H}{r - \Delta V - c_L}\right] \alpha \Delta V - D. \tag{20}$$

Similarly, deviation to any price $\widehat{p}$ satisfying (18) is strictly gainful for an L type firm if, and only if,

$$\pi_L^* < \alpha \Delta V \left[\frac{r - c_L}{r - \Delta V - c_L}\right] - D - f. \tag{21}$$

Using (17), one can show that: $p_0 \geq \underline{p}_L + \Delta V$ if, and only if,

$$D \geq f\left[(1+\alpha)\frac{\Delta V}{\Delta c} - 1\right] - \left(\frac{\alpha(1-\alpha)}{2}\right) \Delta V. \tag{22}$$

First, suppose that (22) does not hold so that $r = \underline{p}_L + \Delta V$. In this case, (20) reduces to

$$D < \Delta V - (1 - \frac{\alpha}{2}) \Delta c = \overline{D} \tag{23}$$

and (21) reduces to

$$D < \Delta V - f. \tag{24}$$

It is easy to check that if (22) does not hold, (24) implies (23) so that a pure price signaling equilibrium exists if, and only if, (23) does not hold i.e.,

$$D \geq \overline{D}. \tag{25}$$

Next, suppose that (22) holds so that $r = p_0$. In this case, one can check that both (20) and (21) reduce to

$$D < \frac{\alpha}{2}\Delta V + f[\frac{\Delta V}{\Delta c} - 1]. \tag{26}$$

Thus, if (22) holds then a pure price signaling equilibrium exists if, and only if, (26) does not hold i.e.,

$$f \leq \frac{D - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c}. \tag{27}$$

We are now ready to establish the proposition as stated. First, we argue that if (25) holds, then a pure price signaling equilibrium always exists. If (16) does not hold, then we know that a pure price signaling equilibrium exists. If (16) and (22) do not hold, then we have shown that (25) implies that a pure price signaling equilibrium exists. If (16) holds and (22) does not hold, then one can show that (25) implies (27) so that once again a pure price signaling equilibrium exists. Next, suppose that $D \in [\underline{D}, \overline{D})$. As $f^*(.)$ is increasing, $f \leq f^*(D)$ implies $f \leq \lim_{D \uparrow \overline{D}} f^*(D) = (1 - \frac{\alpha}{2})\Delta c$ and using this,

$$f\left[(1+\alpha)\frac{\Delta V}{\Delta c} - 1\right] - \frac{\alpha(1-\alpha)}{2}\Delta V \leq \frac{\alpha}{2}\Delta V + f[\frac{\Delta V}{\Delta c} - 1]. \tag{28}$$

Further, for $D \in [\underline{D}, \overline{D}), f^*(D) = \frac{D - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c}$ so that

$$f \leq f^*(D) \Leftrightarrow D \geq f(\frac{\Delta V}{\Delta c} - 1) + \frac{\alpha}{2}\Delta V. \tag{29}$$

Combining (28) and (29), we see that (22) and (27) hold so that a pure price signaling equilibrium exists (whether or not (16) holds). On the other hand, if $f > f^*(D)$, then $f > f^*(D) \geq f^*(\underline{D}) = \frac{\alpha}{2}\Delta c$ and further from (29), $D < f(\frac{\Delta V}{\Delta c} - 1) + \frac{\alpha}{2}\Delta V$ which together implies that (16) does not hold. As $D < \overline{D}$ and further, $f > f^*(D) = \frac{D - \frac{\alpha}{2}\Delta V}{\Delta V - \Delta c}$ implies (27) does not hold, a pure price signaling equilibrium does not exist when (16) does not hold. Finally, consider $D \in (0, \underline{D})$. If $f \leq f^*(D) = \frac{D}{2\frac{\Delta V}{\Delta c} - 1}$, then (16) holds so that a pure signaling equilibrium exists. Now, consider $f > f^*(D)$. Then, (16) does not hold. As $D < \underline{D} < \overline{D}$, a pure price signaling equilibrium does not exist if (22) does not hold. On the other hand, using $D < \underline{D} = \alpha(\Delta V - \frac{\Delta c}{2})$ one can show that

$$f^*(D) = \frac{D}{2\frac{\Delta V}{\Delta c} - 1} > \frac{D - \frac{\alpha}{2}\Delta V}{\frac{\Delta V}{\Delta c} - 1}$$

so that $f > f^*(D)$ implies (27) does not hold. Thus, a symmetric pure price signaling equilibrium does not exist for $f > f^*(D)$.

Proof of Proposition 3.

Given the arguments outlined in the main text preceding the proposition, it remains to rule out any incentive to deviate to an out-of-equilibrium price $\widehat{p} \in (\overline{p}_L^D, p_H^D)$ without disclosing and this requires that out-of-equilibrium beliefs assign sufficiently high probability to the deviating firm being of L-type Using an argument similar to that at the beginning of the proof of Proposition 3, such a belief is consistent with the D1 criterion if, and only if,

$$\frac{p_H^D - c_L}{p_H^D - c_H} \geq \frac{\pi_L^*}{\pi_H^*} = \frac{\alpha(p_H^D - \Delta V - c_L)}{\frac{\alpha}{2}(p_H^D - c_H) - D}.$$

which reduces to

$$f\left(2\Delta V - \Delta c - \frac{2\,(D+f)}{\alpha}\right) \geq D\Delta c$$

i.e., condition (6).

Proof of Proposition 4.

To deter an $H-$type from deviating to not disclosing, out-of-equilibrium beliefs must regard any such deviating firm as low quality with sufficiently high probability, and this can be shown to be consistent with the D1 criterion as $L-$types earn zero profit in this equilibrium. Following the arguments outlined in the main text preceding the proposition, it remains to show that an $L-$type firm has no strict incentive to disclose (falsely) and charge the lower bound of the support of an H-type's equilibrium prices. First, consider the equilibrium where the H-type's price distribution has no mass point; in this case, the distribution function F of the $H-$type's price satisfies:

$$(p_H - c_H)[(1-\alpha) + \alpha(1 - F(p_H)] = (\Delta V - \Delta c)(1-\alpha), p_H \in [\underline{p}_H^D, \overline{p}_H^D] \tag{30}$$

where $\underline{p}_H^D = (\Delta V - \Delta c)(1-\alpha) + c_H$. If an $L-$type firm discloses (falsely) and charges $\underline{p}_H^D$ (the optimal deviation price in the interval $[\underline{p}_H^D, \overline{p}_H^D]$), then his deviation profit equals

$$\underline{p}_H^D - c_L - (D+f) = \Delta V(1-\alpha) + \alpha\Delta c - (D+f)$$

so that the deviation is not gainful if, and only if,

$$D + f \geq \Delta V(1-\alpha) + \alpha \Delta c. \tag{31}$$

Under (31), an $L-$type firm will also not gain by disclosing and deviating to a price less than $\underline{p}_H^D$ even if it is thought of as a high quality firm with probability one. It is easy to check that no other deviation is gainful (with sufficiently pessimistic belief).

Next, consider the equilibrium where an H-type's price distribution has a mass point. From the first proposition in the online Appendix, the mass point can only be at the lower bound of its price distribution and this lower bound price must be an isolated point in the support. In particular, $H-$types randomize over prices in the interval $[\underline{p}_H^D, c_L + \Delta V)$ with some probability κ, and with probability $1-\kappa$, they charge a deterministic price equal to $\widehat{p}_H^D < \underline{p}_H^D$. As the H-type must be indifferent between prices in its equilibrium strategy, it follows that

$$\underline{p}_H^D = c_H + \frac{1-\alpha}{1-\alpha+\alpha\kappa}(\Delta V - \Delta c), \widehat{p}_H^D = c_H + \frac{1-\alpha}{1-\alpha+\alpha\frac{1+\kappa}{2}}(\Delta V - \Delta c).$$

Using part (a) of the first proposition in the online Appendix, $L-$types must be indifferent between their equilibrium strategy and imitating $\widehat{p}_H$ with disclosure which yields

$$\left(1-\alpha+\alpha\frac{1+\kappa}{2}\right)\Delta c + (1-\alpha)(\Delta V - \Delta c) - D - f = 0,$$

so that

$$\kappa = \frac{2\left(D + f - (1-\alpha)\Delta V - \frac{\alpha}{2}\Delta c\right)}{\alpha \Delta c}.$$

$\kappa \in [0,1]$ if, and only if,

$$(1-\alpha)\Delta V + \alpha\Delta c \geq D + f \geq [(1-\alpha)\Delta V + \frac{\alpha}{2}\Delta c]. \tag{32}$$

As the $H-$type is indifferent over prices in the range $[\underline{p}_H^D, c_L + \Delta V)$ and the price $\widehat{p}_H$, it is easy to show that the $L-$type (with lower marginal cost) cannot gain by deviating to a price in the range $[\underline{p}_H, c_L + \Delta V)$. It is easy to check now that no other deviation can be gainful for either type. Combining (31) and (32) we obtain condition (8) as one of the two necessary and sufficient conditions for this equilibrium to exist (the other condition being (7)).

Proof of Proposition 7.

In a fully distortionary partial disclosure equilibrium, a high quality type does not sell in the state where its rival has low quality. It follows then that $\underline{p}_H^D \geq \overline{p}_L + \Delta V$. Suppose the disclosing $H-$type randomizes over prices with distribution function F_H. Then it cannot have a probability mass point at $\overline{p}_H^D$ (see Proposition 2(d)) and $\overline{p}_H^D \leq p_H^{ND}$, it sells to all buyers with probability one in the state where the rival firm is also of high quality but does not disclose. Further, $\overline{p}_H^D > \underline{p}_H^D \geq \overline{p}_L + \Delta V$. A disclosing $H-$type's profit at any $p \in (\overline{p}_L + \Delta V, \overline{p}_H^D)$ is

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1 - F_H(p))]\,(p - c_H) - D = \alpha(1-\gamma_H)(\overline{p}_H^D - c_H) - D, \tag{33}$$

while setting a price equal to $p - \Delta V > \overline{p}_L$ the low quality firm would make a profit of

$$[\alpha(1-\gamma_H) + \alpha\gamma_H(1 - F_H(p))]\,(p - \Delta V - c_L),$$

which using (33) can be rewritten as

$$\alpha(1-\gamma_H)(\overline{p}_H^D - c_H)\frac{p - \Delta V - c_L}{\overline{p}_H^D - c_H},$$

and this is increasing in p, implying a low quality firm would gain by deviating to prices larger than $\overline{p}_L$, a contradiction. Thus, the disclosing high quality type must charge a deterministic price p_H^D. There are two possibilities: (a) $p_H^D < p_H^{ND}$ and all buyers buy from the disclosing high quality type when both firms are of high quality and only one discloses, or (b) $p_H^D = p_H^{ND} = p_H$ (say) and buyers randomize between the disclosing and non-disclosing firms when both are high quality types and only one discloses.

First, we derive the necessary and sufficient condition for an equilibrium of type (b). Using Proposition 2, at price p_H, H types sell only if the rival firm is a high quality type and $p_H \geq \overline{p}_L + \Delta V$. If $p_H > \overline{p}_L + \Delta V$, an L-type will always earn strictly higher profit by deviating to a price slightly higher than $\overline{p}_L$. Therefore, $p_H = \overline{p}_L + \Delta V$. As this is a symmetric equilibrium, buyers buy from each firm with equal probability when they both disclose or when neither discloses. When both firms charge p_H and only one discloses, let $\widetilde{\beta} \in (0,1)$ denote the probability that each buyer buys from the disclosing firm in that state (i.e., the expected market share of the disclosing firm is $\widetilde{\beta}$). L types should randomize over the interval $[\underline{p}_L, \overline{p}_L]$, with $p_H = \overline{p}_L + \Delta V$ and no mass points; it is easy to check that $\underline{p}_L = \alpha\overline{p}_L + (1-\alpha)c_L$ and the equilibrium profits are given by

$$\pi_H^* = \alpha(\frac{\gamma_H}{2} + \widetilde{\beta}(1-\gamma_H))(p_H - c_H) - D = \alpha(\frac{1-\gamma_H}{2} + \gamma_H(1-\widetilde{\beta}))(p_H - c_H) \tag{34}$$

and

$$\pi_L^* = \alpha(p_H - \Delta V - c_L), \tag{35}$$

respectively. As the D1 refinement requires the $L-$type to be indifferent between its equilibrium strategy and and setting p_H with and without disclosure, we have

$$\pi_L^* = \alpha(\frac{\gamma_H}{2} + \beta(1-\gamma_H))(p_H - c_L) - D - f = \alpha(\frac{1-\gamma_H}{2} + \gamma_H(1-\beta))(p_H - c_L). \quad (36)$$

Using this in (34), we get

$$\widetilde{\beta} = \frac{1}{2} + \frac{f}{\alpha \Delta c}. \quad (37)$$

Note that $\widetilde{\beta} > 0.5$. Further, $\widetilde{\beta} \leq 1$ if, and only if,

$$f \leq \frac{\alpha}{2}\Delta c. \quad (38)$$

From (36) and (37) we have $p_H = c_L + \Delta c(\frac{D+f}{f}) = c_H + \frac{D\Delta c}{f}$ and the $H-$type makes strictly positive profit. Substituting the expressions for $\widetilde{\beta}$ and p_H into the first equation in (36) and using (35) we have $\gamma_H = \alpha\frac{\Delta V}{D+f} - \frac{\alpha \Delta c}{2f}$.so that $0 < \gamma_H < 1$ if, and only if,

$$f > \frac{D}{2\frac{\Delta V}{\Delta c} - 1} \quad (39)$$

and

$$D > f\left(\frac{2\Delta V}{\Delta c + \frac{2f}{\alpha}} - 1\right). \quad (40)$$

Note that (40) is equivalent to $D > \widetilde{D}(f)$under constraint (38). Further, $\frac{D}{2\frac{\Delta V}{\Delta c}-1} \geq \frac{\alpha}{2}\Delta c$ for $D \geq \frac{\alpha}{2}\Delta c(2\frac{\Delta V}{\Delta c} - 1) = \alpha(\Delta V - \Delta c)$ so that (39) and (38) can be jointly satisfied only if $D < \alpha(\Delta V - \frac{\Delta c}{2}) = \underline{D}$ and for this range of D, (39) is equivalent to $D < D^*(f)$. Thus, $0 < f \leq \frac{\alpha}{2}\Delta c$, and $\widetilde{D}(f) < D < D^*(f)$ are necessary and sufficient for the existence of an equilibrium of type (b).

In an equilibrium of type (a), the equilibrium profits are given by

$$\pi_H^* = \alpha(\frac{\gamma_H}{2} + (1-\gamma_H))(p_H^D - c_H) - D = \alpha(\frac{1-\gamma_H}{2})(p_H^{ND} - c_H)$$

$$\pi_L^* = \alpha(p_H^D - \Delta V - c_L).$$

As the D1 refinement requires the $L-$type to be indifferent between its equilibrium strategy and setting p_H with and without disclosure, we have

$$\pi_L^* = \alpha(\frac{\gamma_H}{2} + (1-\gamma_H))(p_H^D - c_L) - D - f = \alpha(\frac{1-\gamma_H}{2})(p_H^{ND} - c_L),$$

and it is easy to verify that these conditions can be jointly satisfied only if $f = \frac{\Delta c \alpha}{2}$ and $\widetilde{D}(f) < D < D^*(f) = \underline{D}$ at that specific value of f. This completes the proof.

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